Joint dose minimization and variance optimization
for fluence-modulated proton CT

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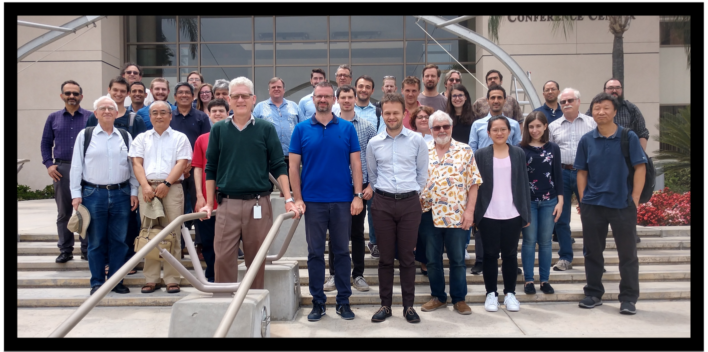
★equal contribution



~~Loma Linda, CA~~
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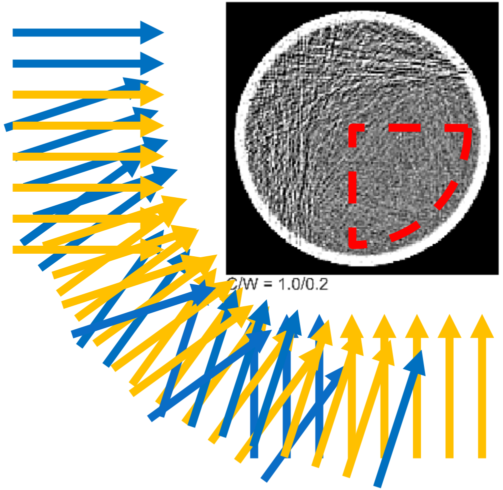
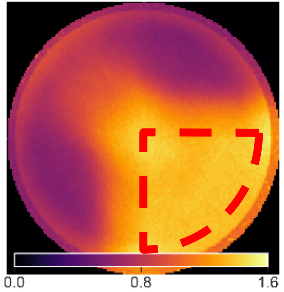
Fluence-modulated proton CT (FMpCT)

Aim: to use **modulated pencil beams for** achieving arbitrary **image noise targets** with FMpCT.

RSP
(relative stopping power)

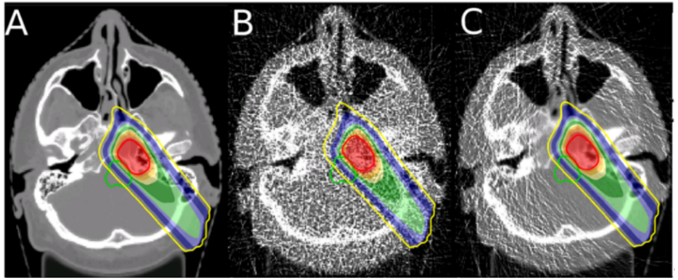


Dose / mGy



Dickmann et al. (2020), Med. Phys., 47, 4
Dickmann et al. (2020), PMB, in press

Motivation: frequent imaging for particle therapy



noise in ROI	Low	High	Low
dose outside ROI	High	Low	Low

Dedes et al. (2017), PMB, 62, 6026

What is left to do?

- Only focus on variance and only indirect handle for dose
- Include dose in optimization to further improve results

Fluence-modulated proton CT (FMpCT)

Analytical reconstruction → Optimization algorithm → Results

Variance reconstruction

Standard reconstruction

projection

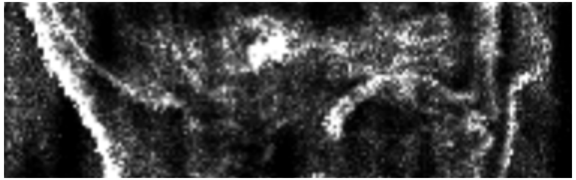
ramp filter → backprojection

volume

$$f(x,y) = \frac{\pi \Delta \xi}{N_P} \sum_{n=1}^{N_P} \{k \circledast p\}(x\cos(\gamma_n) + y\sin(\gamma_n))$$

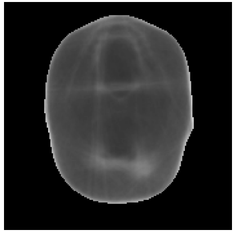
$$\mathrm{Var}[f(x,y)] = f_{\mathrm{interp}} \left(\frac{\pi \Delta \xi}{N_P}\right)^2 \sum_{n=1}^{N_P} \{k^2 \circledast \mathrm{Var}[p]\}(x\cos(\gamma_n) + y\sin(\gamma_n))$$

Variance reconstruction



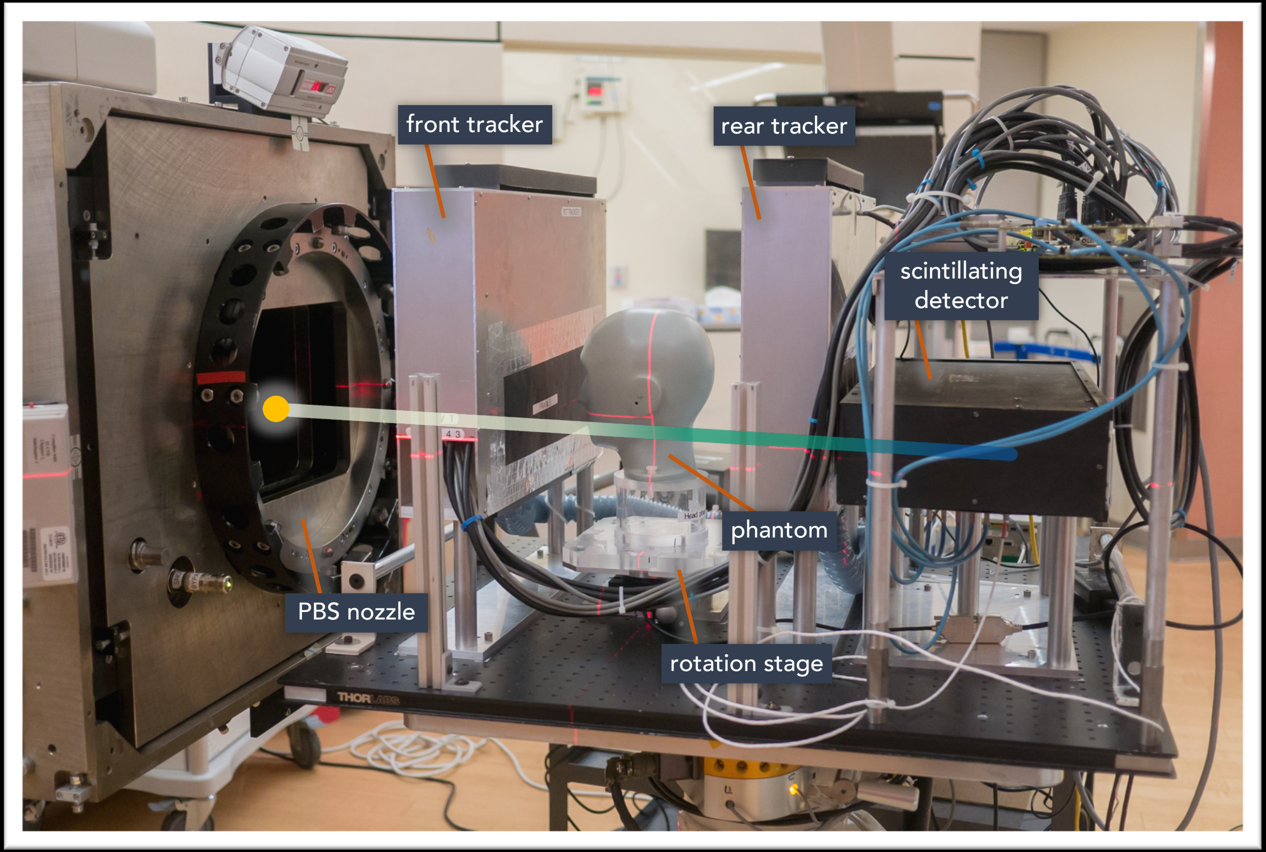
variance projection

(filter) → backprojection



variance volume

Proton computed tomography

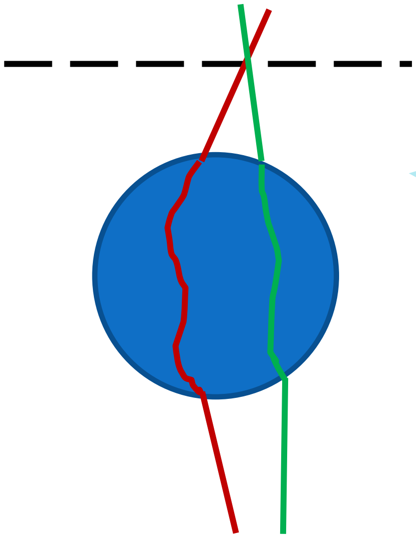


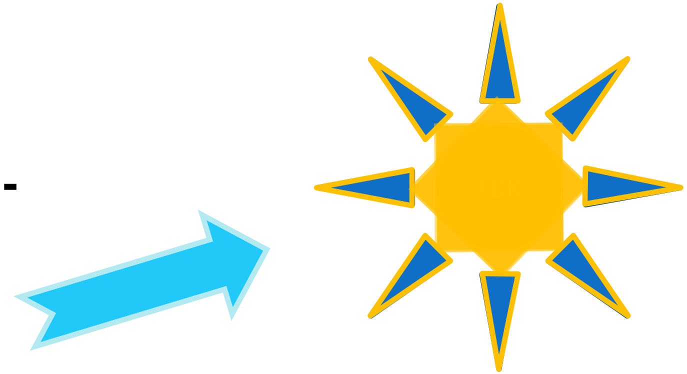
Johnson et al. (2016), IEEE, 63, 1

Bashkirov et al. (2016), Med. Phys., 43, 2

Distance-driven binning

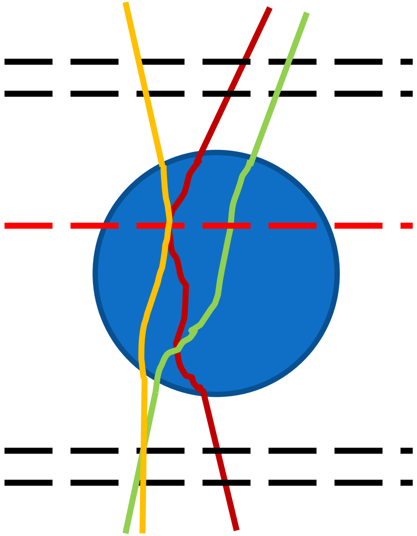
Rear tracker binning

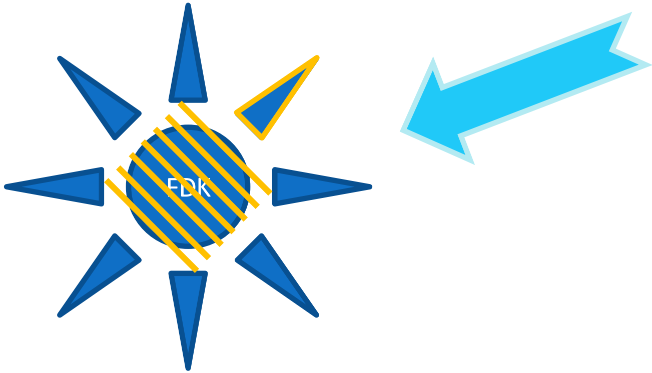




Distance-driven binning

S. Rit et al. (2013), Med. Phys., 40, 3



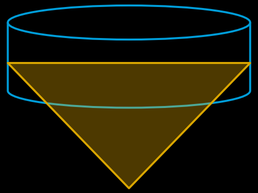


Key information: WEPL and variance information is available at any distance between the two trackers!

Distance-driven binning

Rear-tracker binning

Distance-driven binning

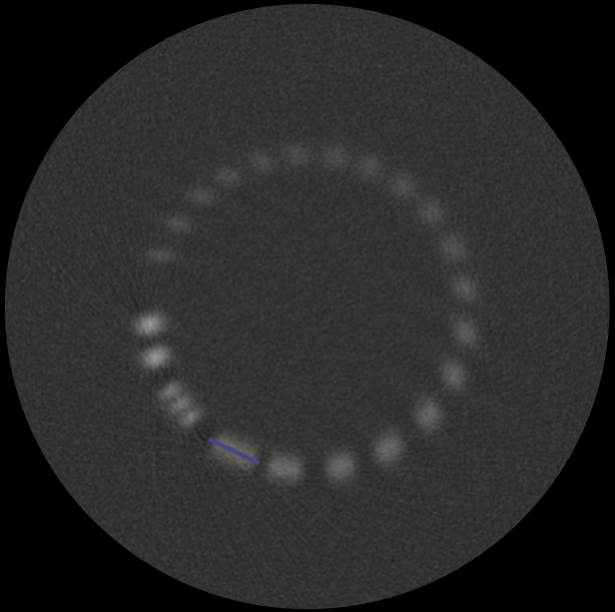


own data

S. Rit et al. (2013), Med. Phys., 40, 3

Distance-driven binning

Rear-tracker binning



Distance-driven binning

S. Rit et al. (2013), Med. Phys., 40, 3

Dose and variance optimization

STEP I

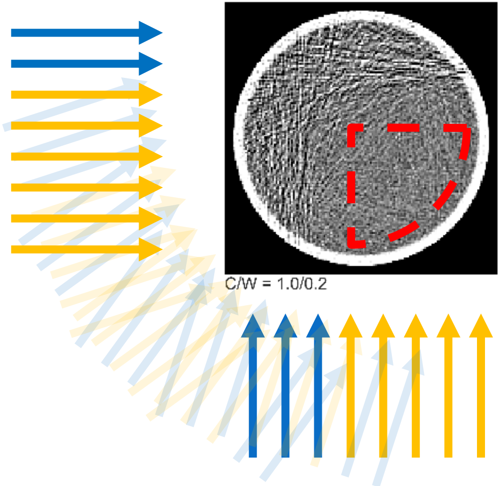
- Forward model for variance and dose

STEP II

- Bixel-wise optimization

STEP III

- Pencil beam optimization



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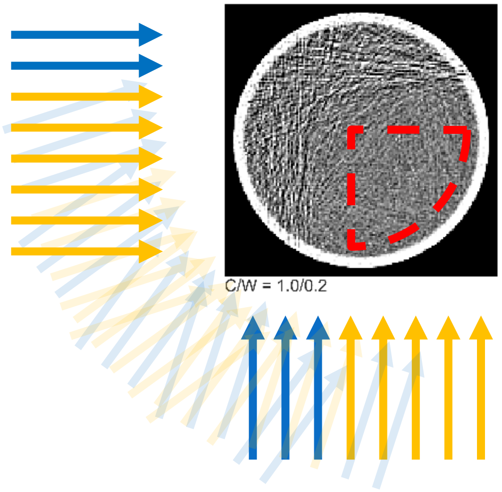
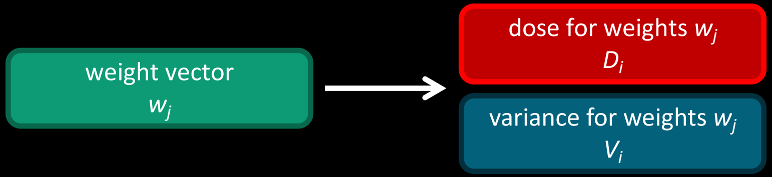


Forward model

- Assume fluence can be modulated in small bixels each associated with a weight w_j



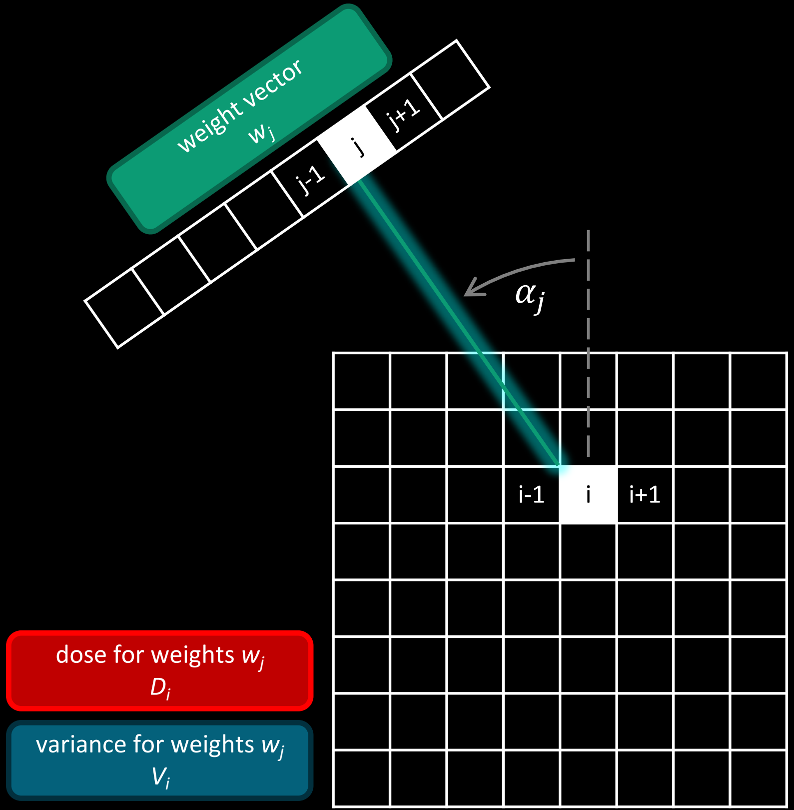
- Formulate problem as matrix multiplication

$$D_i = \sum_{j=1}^{M} D_{ij} \cdot w_j$$

- More difficult for variance

$$V_i = c \cdot \sum_{j=1}^{M} V_{ij} \cdot \frac{1}{w_j} = c \cdot \sum_{j=1}^{M} V_{ij} \cdot \widetilde{w}_j$$

- Because of the inverse dependence, a simple treatment planning approach is not feasible.



Forward model: Dose

- From a Monte Carlo simulation we can get the dose $d_i^{\alpha_j}$ in voxel i at rotation angle α_j for uniform fluence corresponding to $w_j = 1$.
- The dose matrix then is

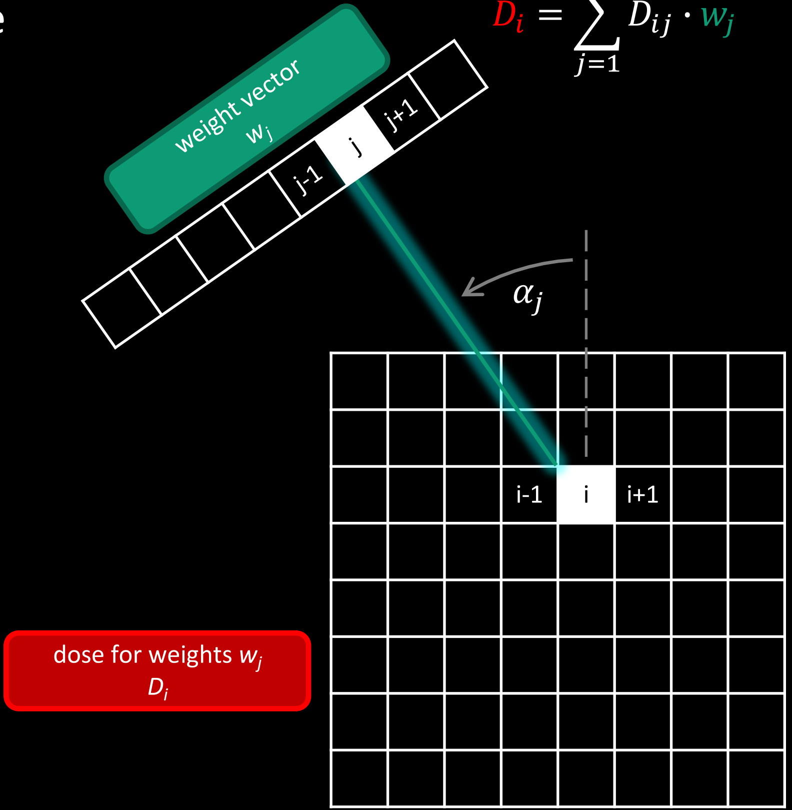
$$D_{ij} = d_i^{\alpha_j} \cdot \delta_{ij}$$

where δ_{ij} is non-zero *"if voxel i corresponds to weight w_j"*.
- A simple implementation could be

$$\delta_{ij} = \begin{cases} 1 & \text{if } x_i \cos\alpha_j + y_i \sin\alpha_j \approx \xi_j \\ 0 & \text{else} \end{cases}$$

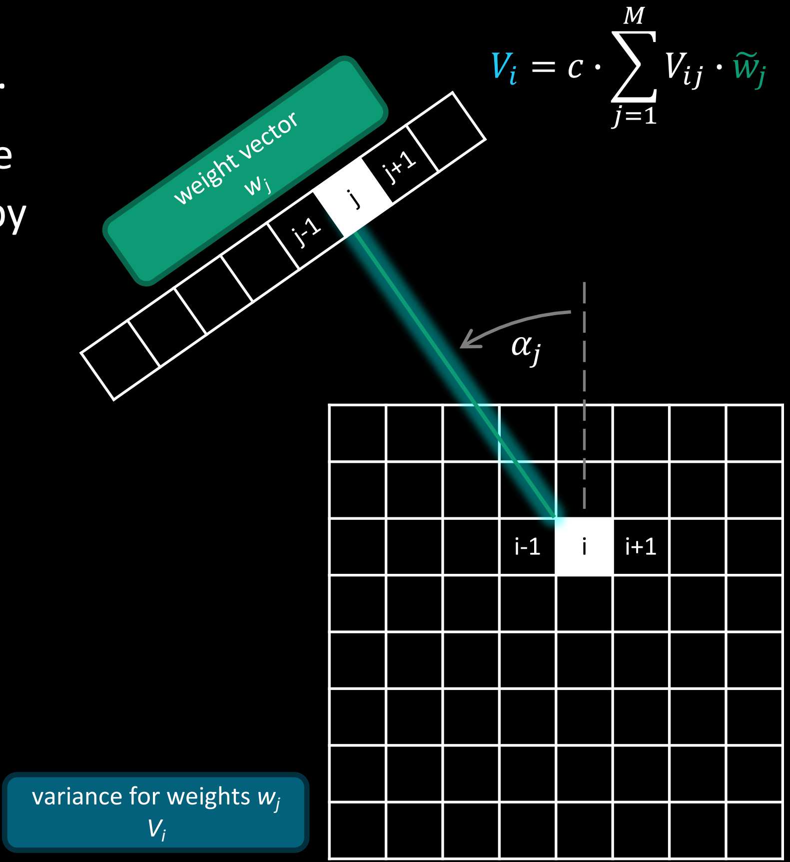
- In fact we perform a linear interpolation between neighboring weights.

$$D_i = \sum_{j=1}^{M} D_{ij} \cdot w_j$$



Forward model: variance

- Variance is proportional to the inverse weights $\tilde{w} = \frac{1}{w_j}$.
- From a Monte Carlo simulation we can get the variance $v_i^{\alpha_j}$ in voxel i at rotation angle α_j for uniform fluence by rotating the distance-driven variance projection.
- The variance matrix then is
$$V_{ij} = v_i^{\alpha_j} \cdot \delta_{ij}$$
where δ_{ij} is defined as for the dose.
- The additional constant is $c = f_{\text{interp}} \cdot \frac{(\pi\Delta\xi)^2}{N_P^2}$, which gives
$$V_i = f_{\text{interp}} \cdot \frac{(\pi\Delta\xi)^2}{N_P^2} \sum_{j=1}^{M} v_i^{\alpha_j} \cdot \delta_{ij} \cdot \frac{1}{w_j}$$
- This is equal to variance reconstruction without filter.

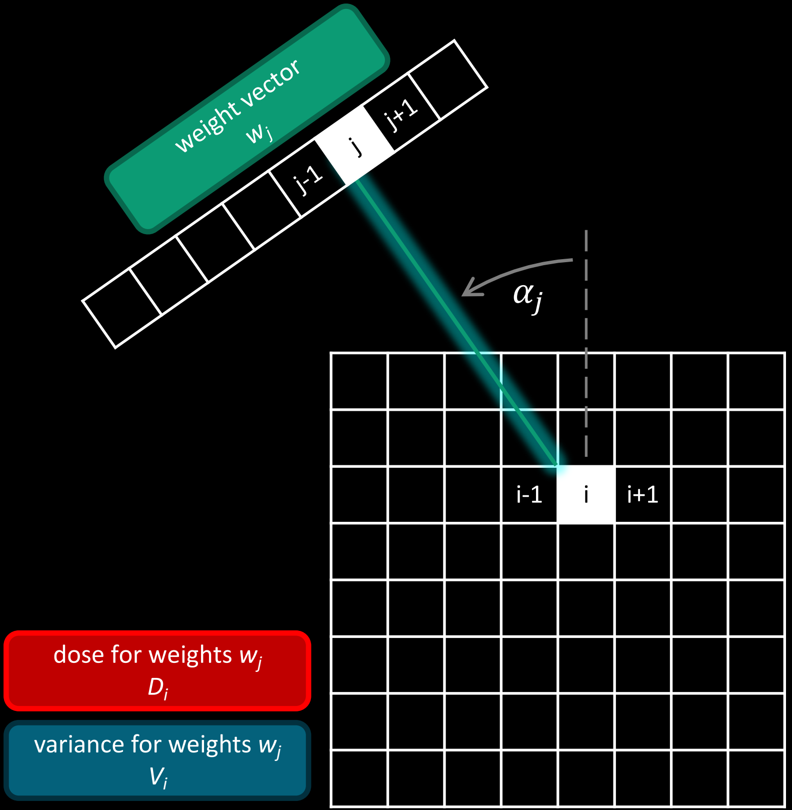


Forward model

weight vector w_j → dose for weights w_j D_i ; variance for weights w_j V_i

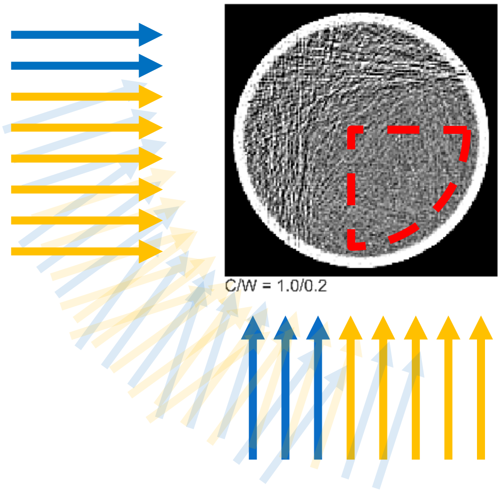
$$D_i = \sum_{j=1}^{M} D_{ij} \cdot w_j$$

$$V_i = c \cdot \sum_{j=1}^{M} V_{ij} \cdot \widetilde{w}_j$$



- Implement forward model as a fast sparse matrix multiplication using Eigen3
  - $N = 60 \times 60 \times 30 = 10^5$ voxels
  - $M = 90 \times 60 \times 30 = 10^5$ bixels
- Use forward model to calculate a joint cost function with a dose and a variance term.

Dose and variance optimization



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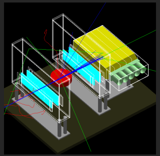
Bixel-wise optimization

Forward model



Monte Carlo simulation

unit variance projections v_i^{aj}

unit dose projections d_i^{aj}

variance matrix V_{ij}

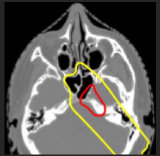
dose matrix D_{ij}

Prescriptions

variance prescription $V_{presc} = 5 \cdot 10^{-4}$

dose minimization $D_{presc} = 0$ mGy

Penalties



ROI and OARs

dose penalty p_i^D

variance penalty p_i^V

Bixel-wise optimization

variance matrix V_{ij}

dose matrix D_{ij}

x

weight vector w_j

Limited memory Broyden Fletcher Goldfarb Shannon minimizer

s.t. $0.05 < w_j < 10$

current variance V_i

current dose D_i

-

variance prescription $V_{presc} = 5 \cdot 10^{-4}$

dose minimization D_{presc} = 0 mGy

$(\ldots)^2$

x

variance penalty p_i^V

dose penalty p_i^D

=

cost function $C(w)$

gradient $\partial/\partial w_j\, C(w)$

$$C(w) = \sum_{i=1}^{N} p_i^D \left(D_i - D_{\text{presc}}\right)^2 + \sum_{i=1}^{N} p_i^V \left(V_i - V_{\text{presc}}\right)^2$$

$$\frac{\partial}{\partial w_j} C(w) = 2 \sum_{i=1}^{N} p_i^D \left(D_i - D_{\text{presc}}\right) D_{ik} - 2 \sum_{i=1}^{N} p_i^V \left(V_i - V_{\text{presc}}\right) \frac{V_{ik}}{w_j^2}$$

Bixel-wise optimization

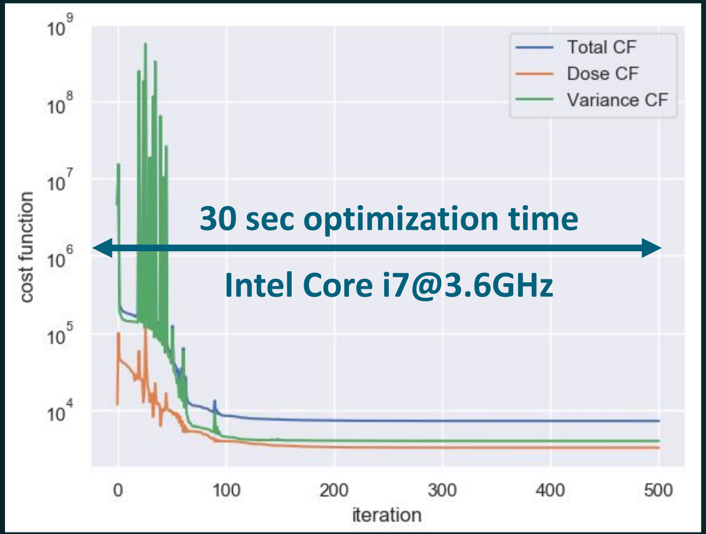
variance matrix
V_{ij}

dose matrix
D_{ij}

x

weight vector
w_j

Limited memory Broyden Fletcher Goldfarb Shannon minimizer
s.t. $0.05 < w_j < 10$



variance prescription
$V_{\text{presc}} = 5 \cdot 10^{-4}$

dose minimization
$D_{\text{presc}} = 0$ mGy

$(\ldots)^2$ x

variance penalty p_i^V

dose penalty p_i^D

=

cost function
$C(w)$

gradient
$\partial/\partial w_j\, C(w)$

$\ldots V_{\text{presc}})^2$

$\frac{\partial}{\partial w \ldots} \ldots (V_i - V_{\text{presc}}) \frac{V_{ik}}{w_j^2}$

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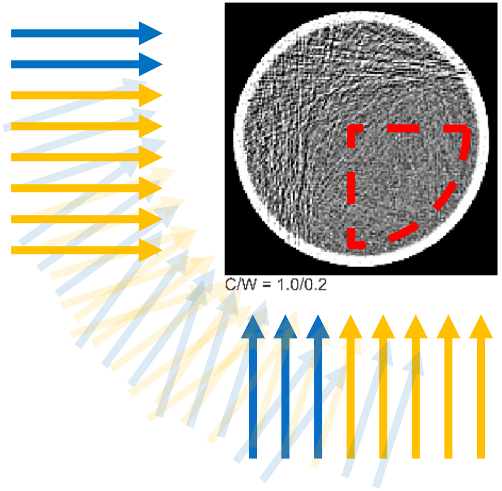
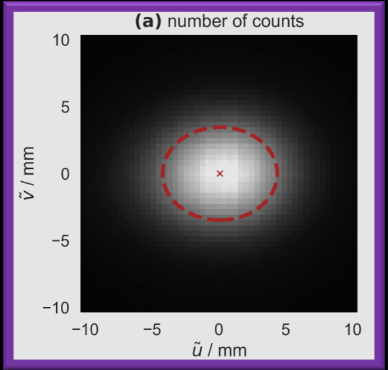


Pencil beam optimization

- Exactly like the D_{ij} matrix, generate an F_{ij} matrix from distance-driven binned proton numbers of a unit fluence scan.
- For the optimized weights w_j, generate modulated fluence projections F_i^{α}
- Fit fluence projections F_i^{α} with an analytical pencil beam model to get pencil beam weights u_k such that

$$F_i^{\alpha} = \sum_{k=1}^{Q} P_{ik} \cdot u_k$$



Dickmann et al. (2020), PMB, in press
Dickmann et al. (2019), PMB, 64, 14

Dose and variance optimization

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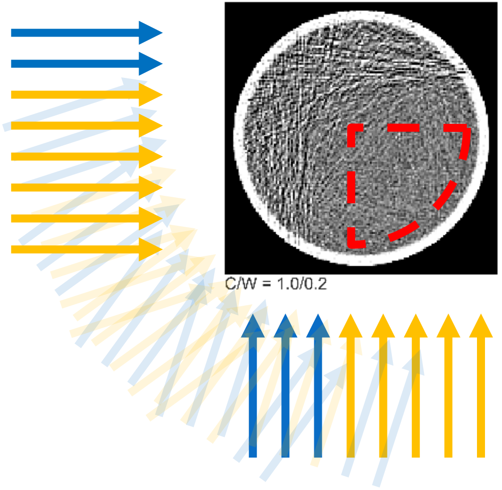
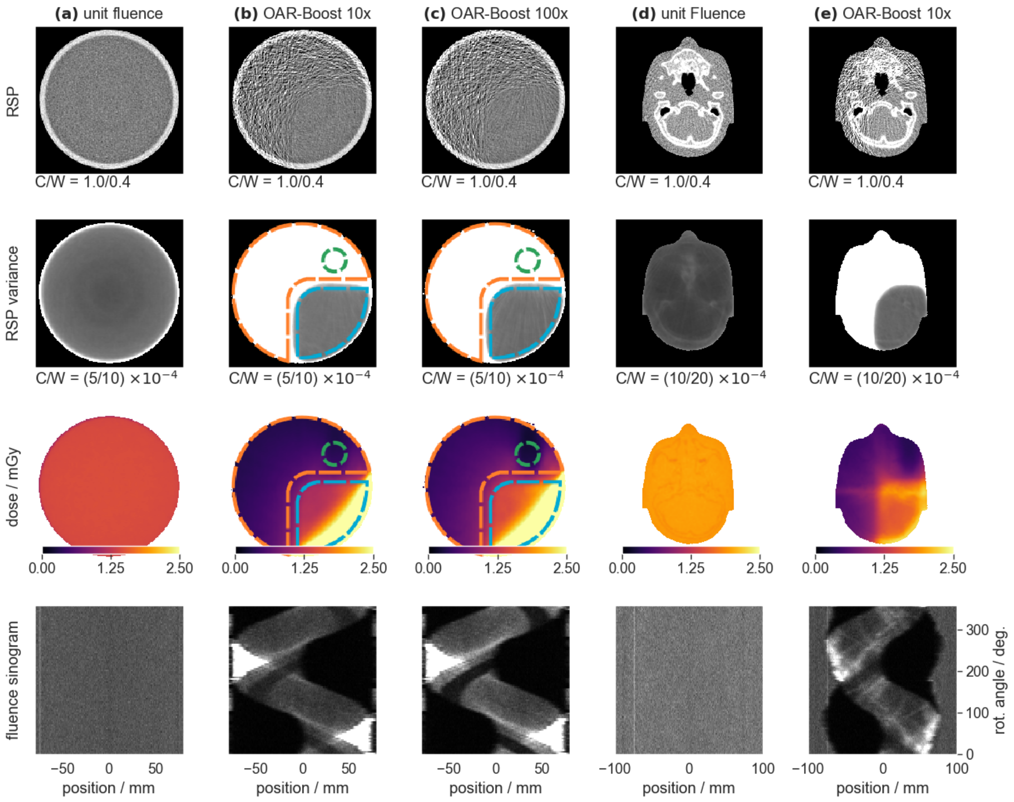
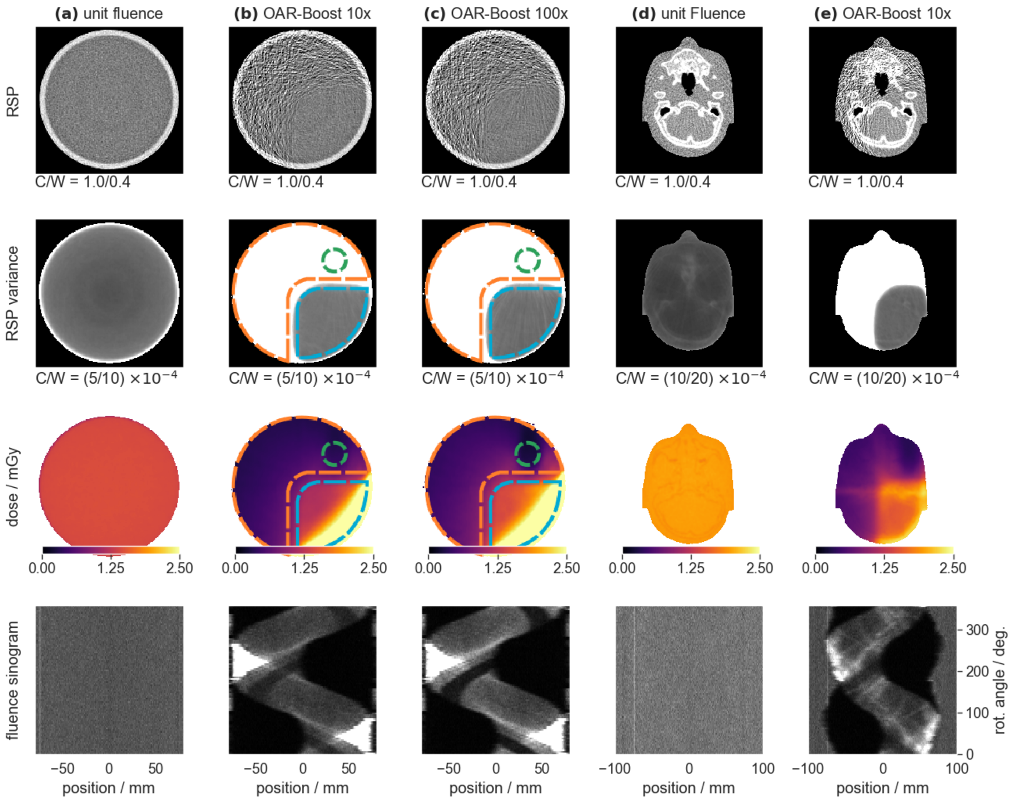


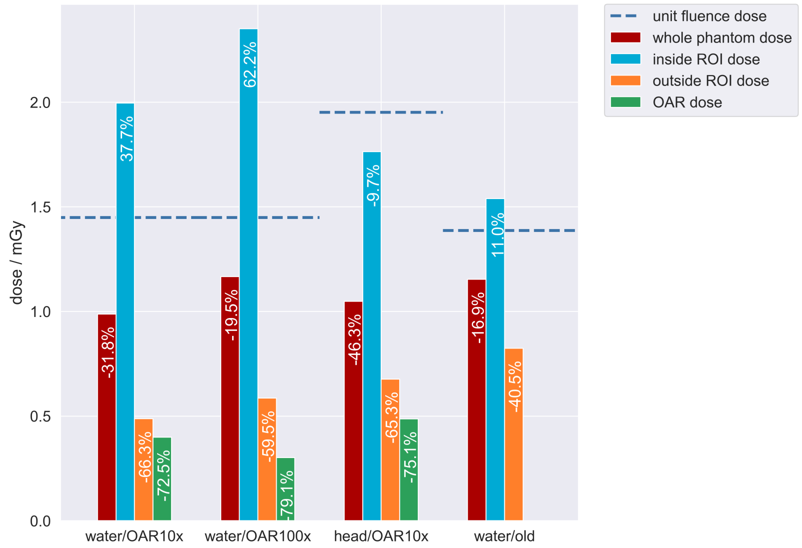
Results



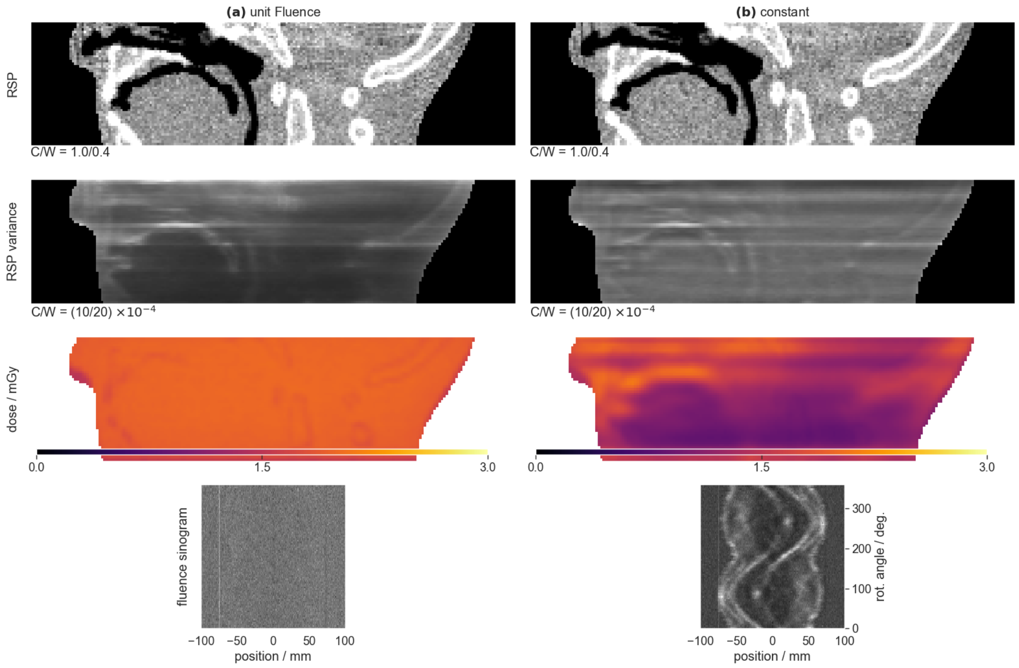
- Variance normalized to 95th percentile value (peak variance) inside the entire phantom
- Constant variance inside the ROI
- Steep increase of variance outside the ROI
- High dose inside the ROI

Results


(a) unit fluence
(b) OAR-Boost 10x
(c) OAR-Boost 100x
(d) unit Fluence
(e) OAR-Boost 10x
RSP
C/W = 1.0/0.4
RSP variance
C/W = (5/10) ×10−4
C/W = (10/20) ×10−4
dose / mGy
0.00
1.25
2.50
fluence sinogram
−50
0
50
−100
100
position / mm
rot. angle / deg.
300
200
100
0


unit fluence dose
whole phantom dose
inside ROI dose
outside ROI dose
OAR dose
dose / mGy
2.0
1.5
1.0
0.5
0.0
37.7%
-31.8%
-66.3%
-72.5%
62.2%
-19.5%
-59.5%
-79.1%
-9.7%
-46.3%
-65.3%
-75.1%
11.0%
-16.9%
-40.5%
water/OAR10x
water/OAR100x
head/OAR10x
water/old

Results



- Dose reduction of 33% over the whole phantom.

Conclusions

I — Fluence-modulated proton CT (FMpCT) can **reduce imaging dose**.

II — The optimization algorithm accounts both for **dose and variance** at the same time.

III — First optimization on **bixels**, then with **pencil beams**.

IV — **Organs-at-risk** can receive an additional dose saving at the cost of increased dose elsewhere.

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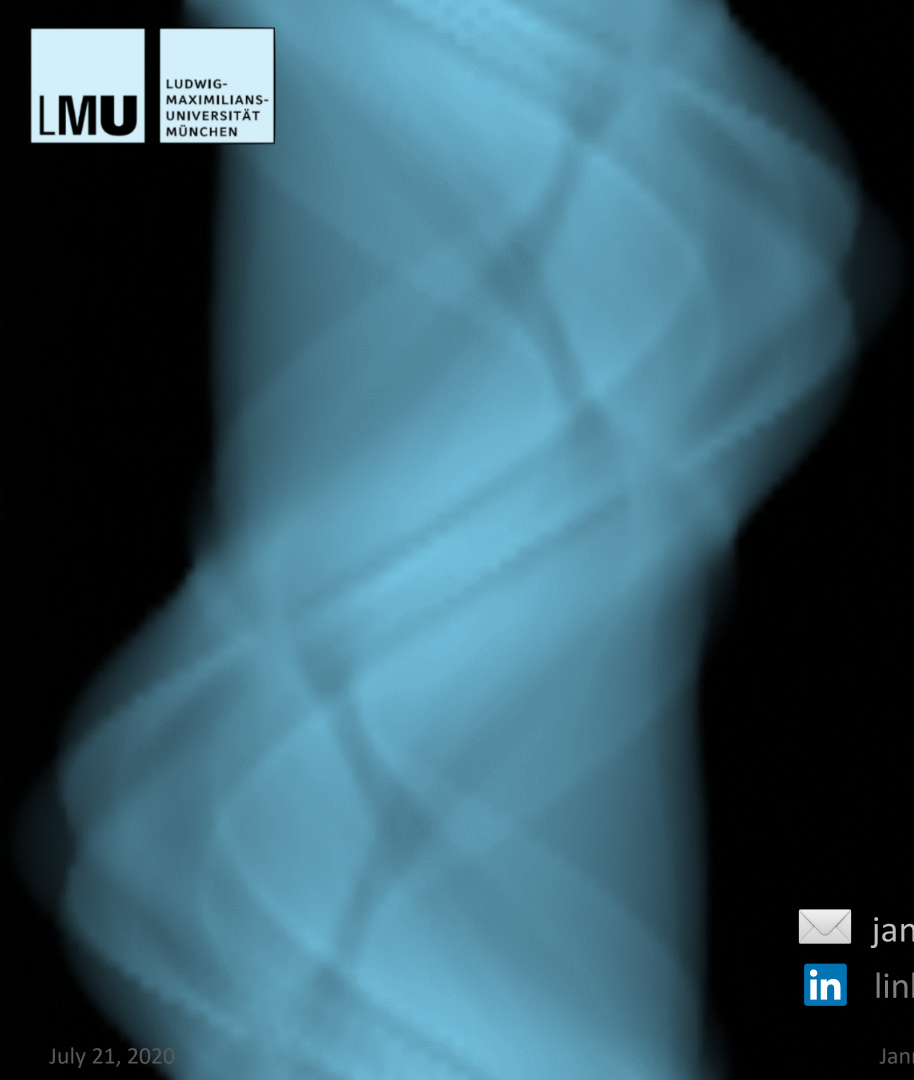
ESTRO



Thank you for your attention!

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