A comprehensive theoretical comparison of proton imaging set-ups in terms of spatial resolution

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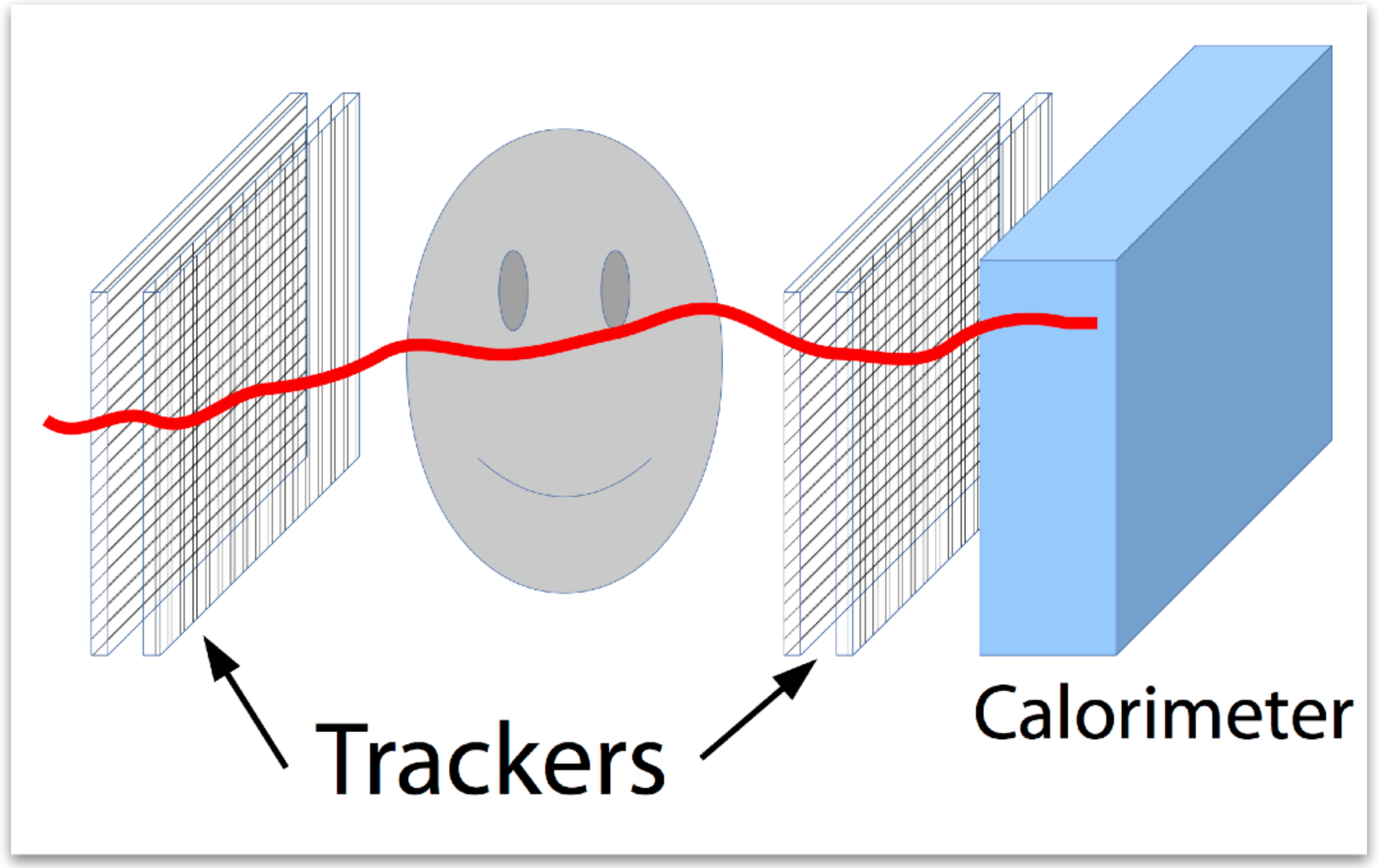
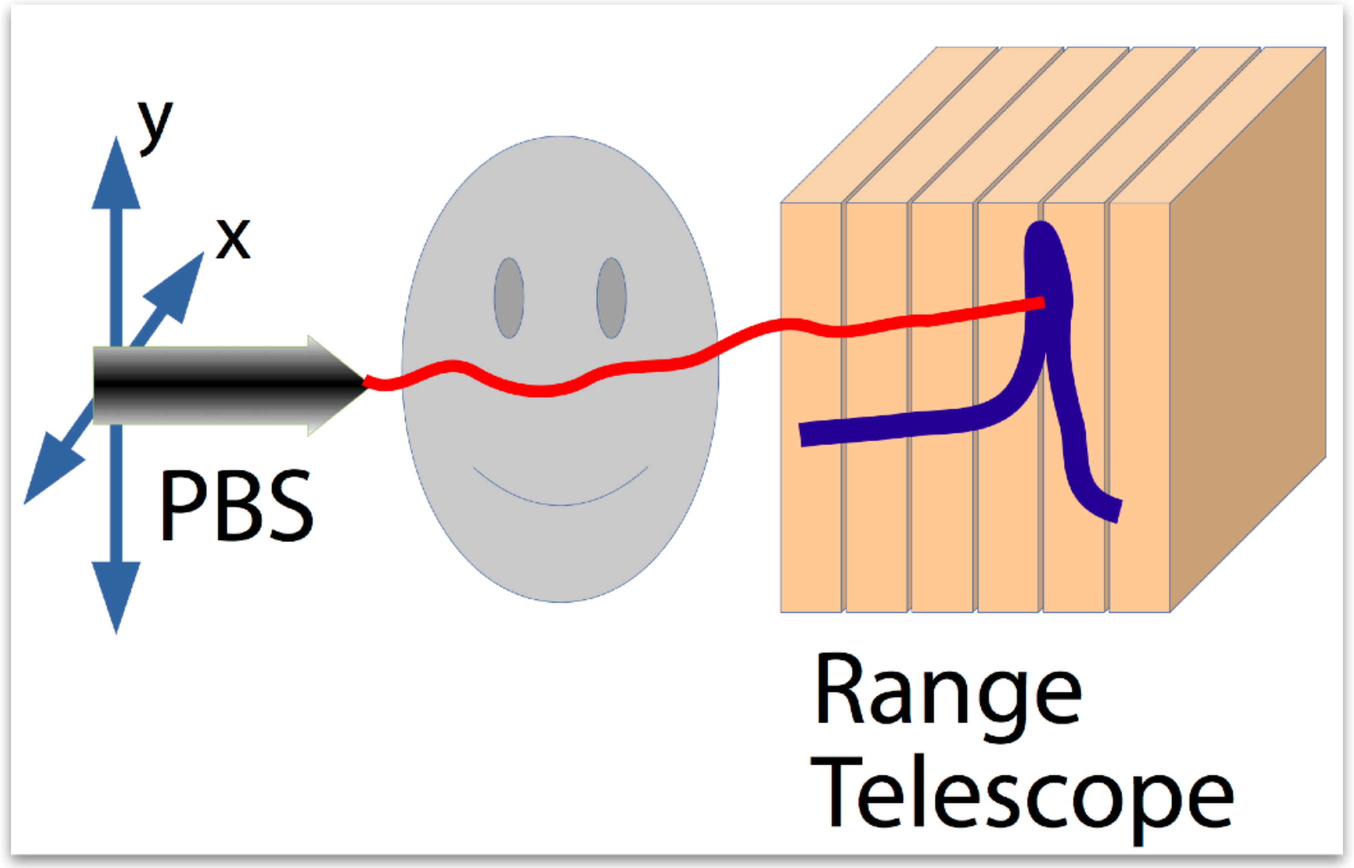
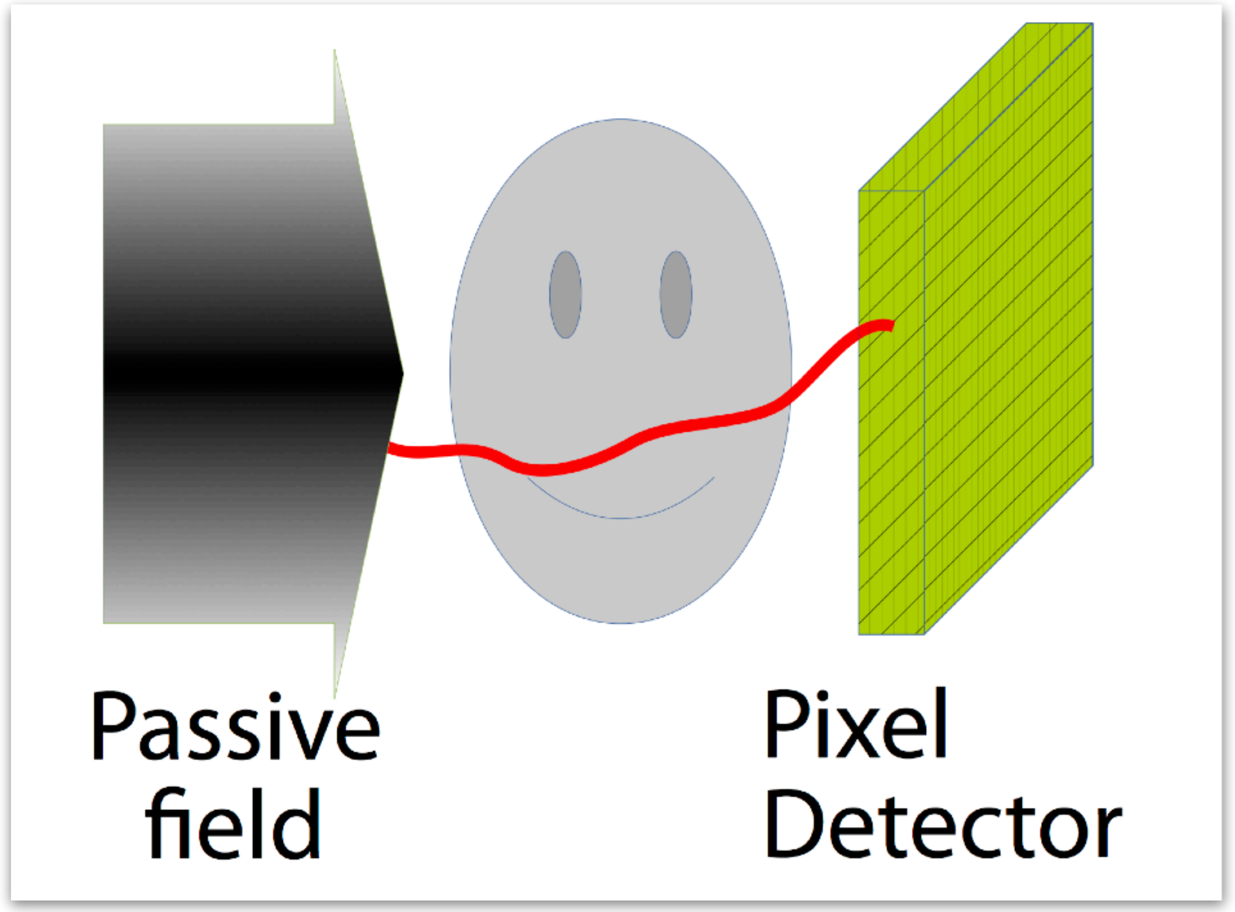
Selection of proton imaging set-ups

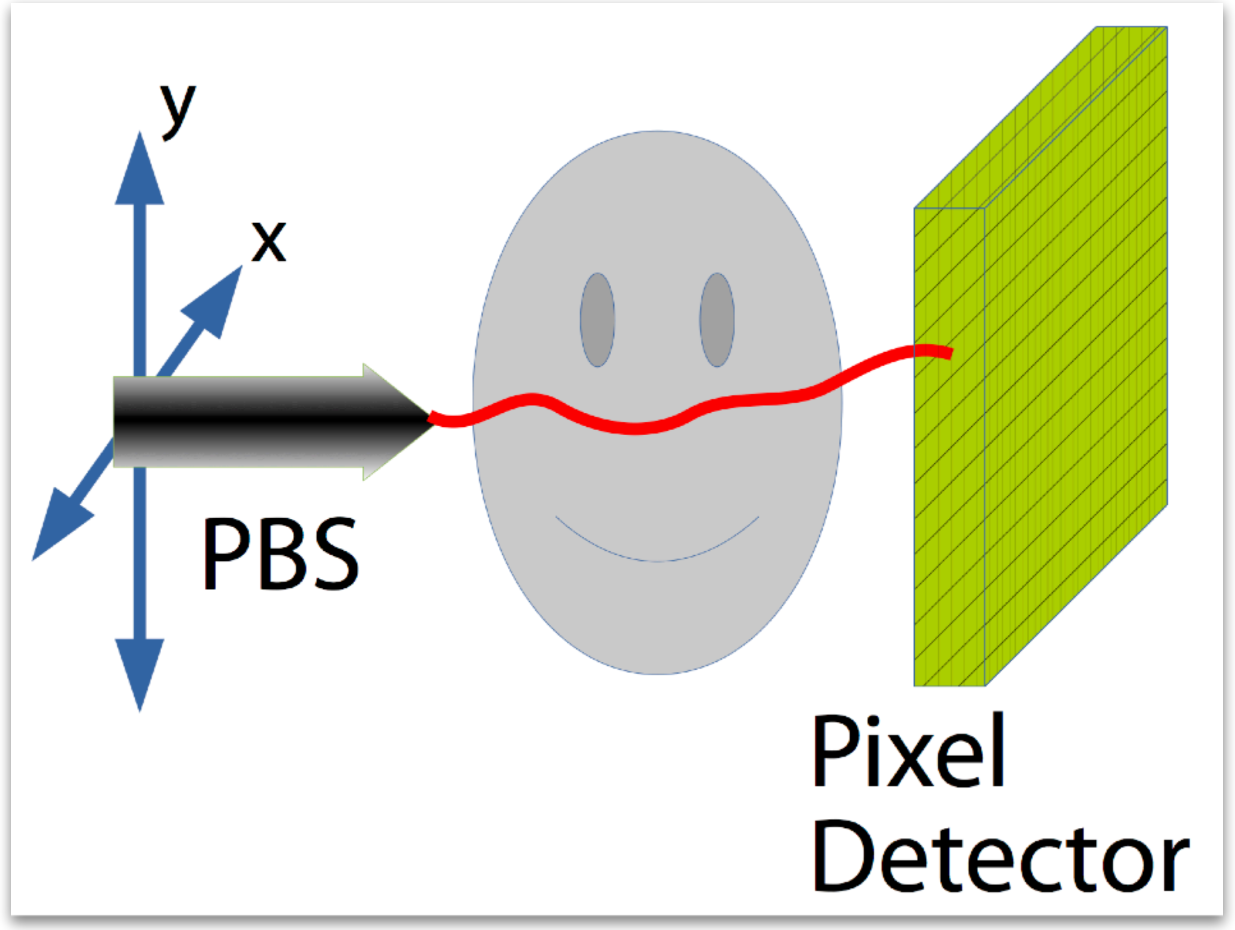




Christian
Ilaria
this
afternoon



Hsiao-Ming
this afternoon



How to choose the set-up?

Cost

Integration into clinical reality

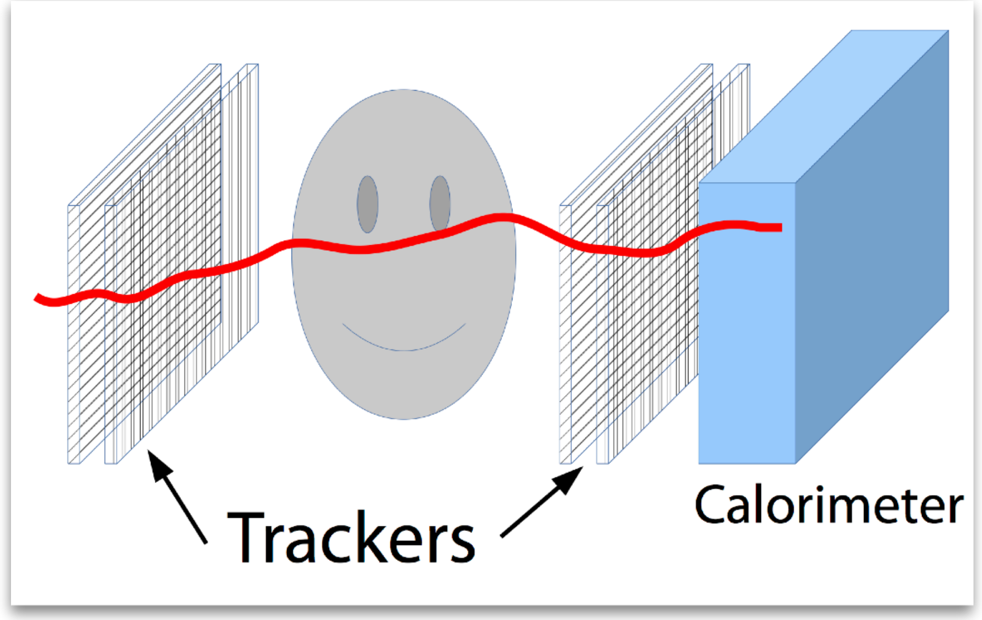
RSP/WET accuracy

Dose to the patient

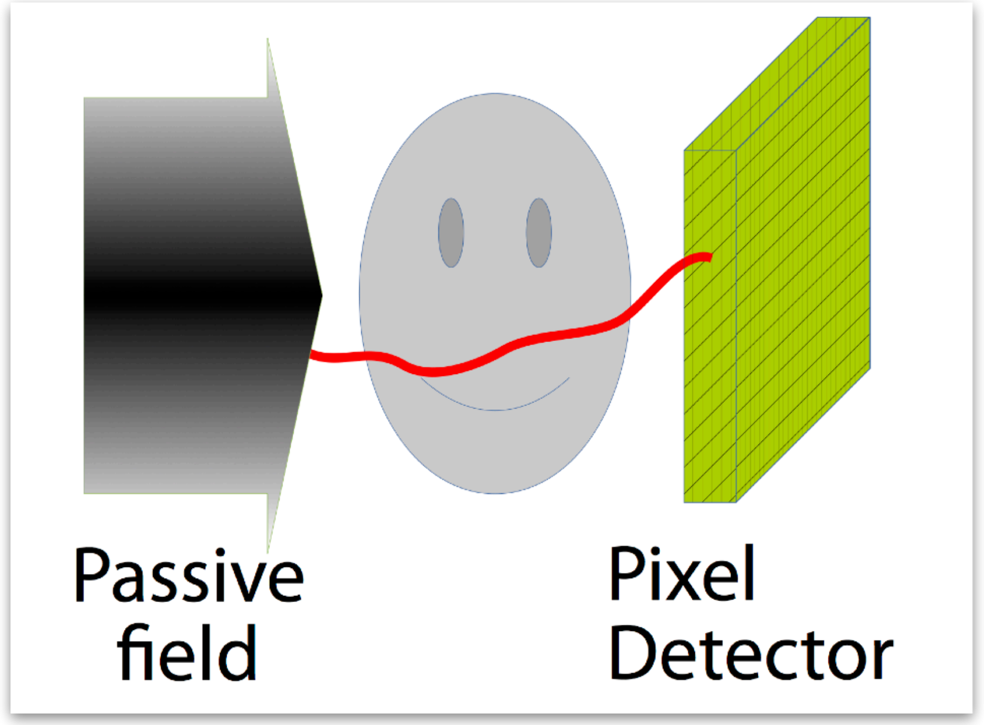
Spatial resolution

Cost

Integration

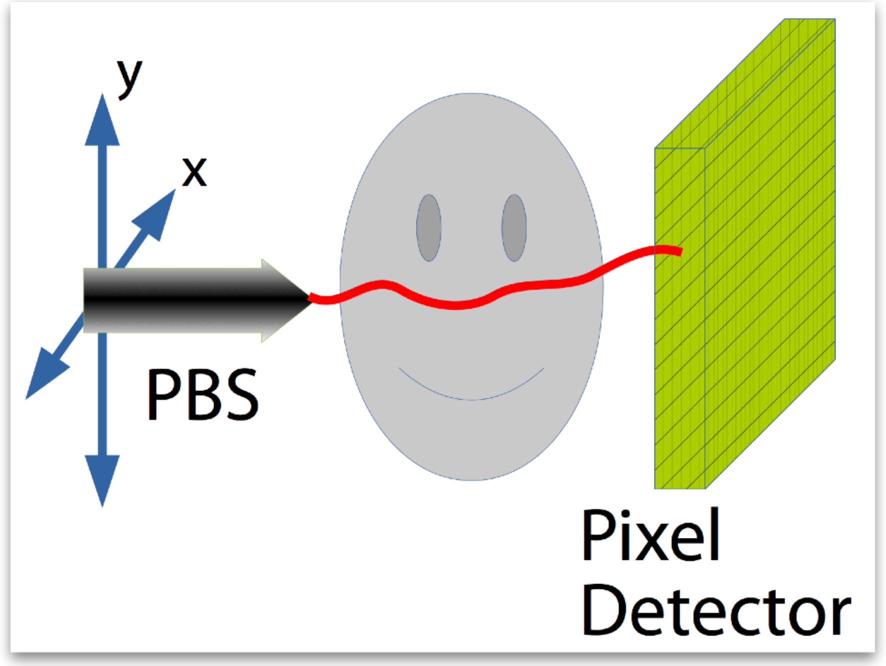


You need to construct:
- **8 tracker devices**
- **1 calorimeter**
- **1 fast electronics**

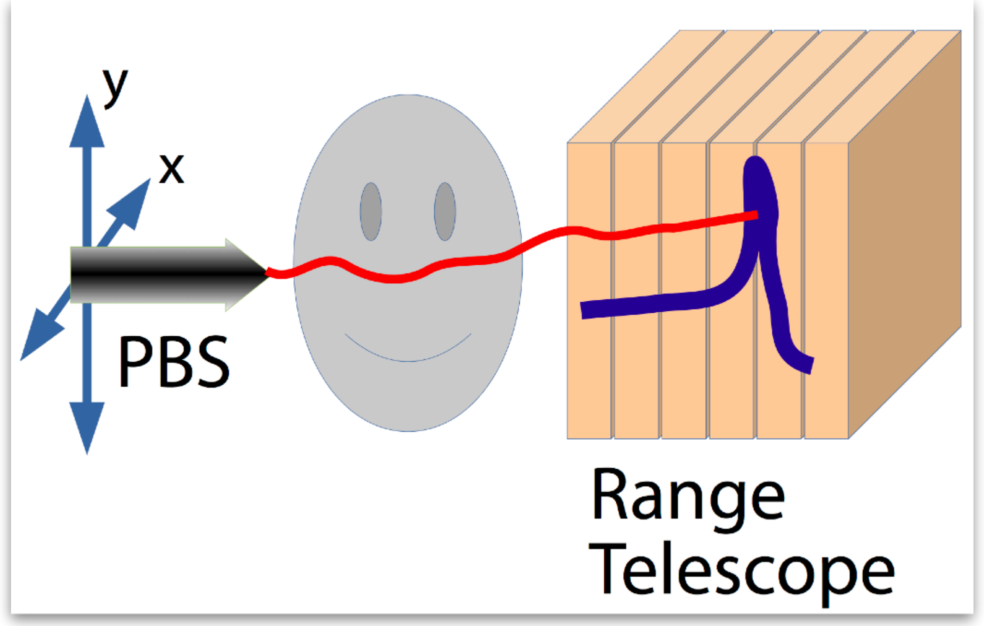


You need to construct:
- **1 Range modulator wheel**

You can re-use:
- **1 X-ray flat panel detector**



You can re-use:
- **1 X-ray flat panel detector**



You can re-use:
- **1 Range telescope**

Simple integration into treatment room

Integration into workflow?

Spatial resolution

Physics in Medicine & Biology

IPEM Institute of Physics and Engineering in Medicine

PAPER

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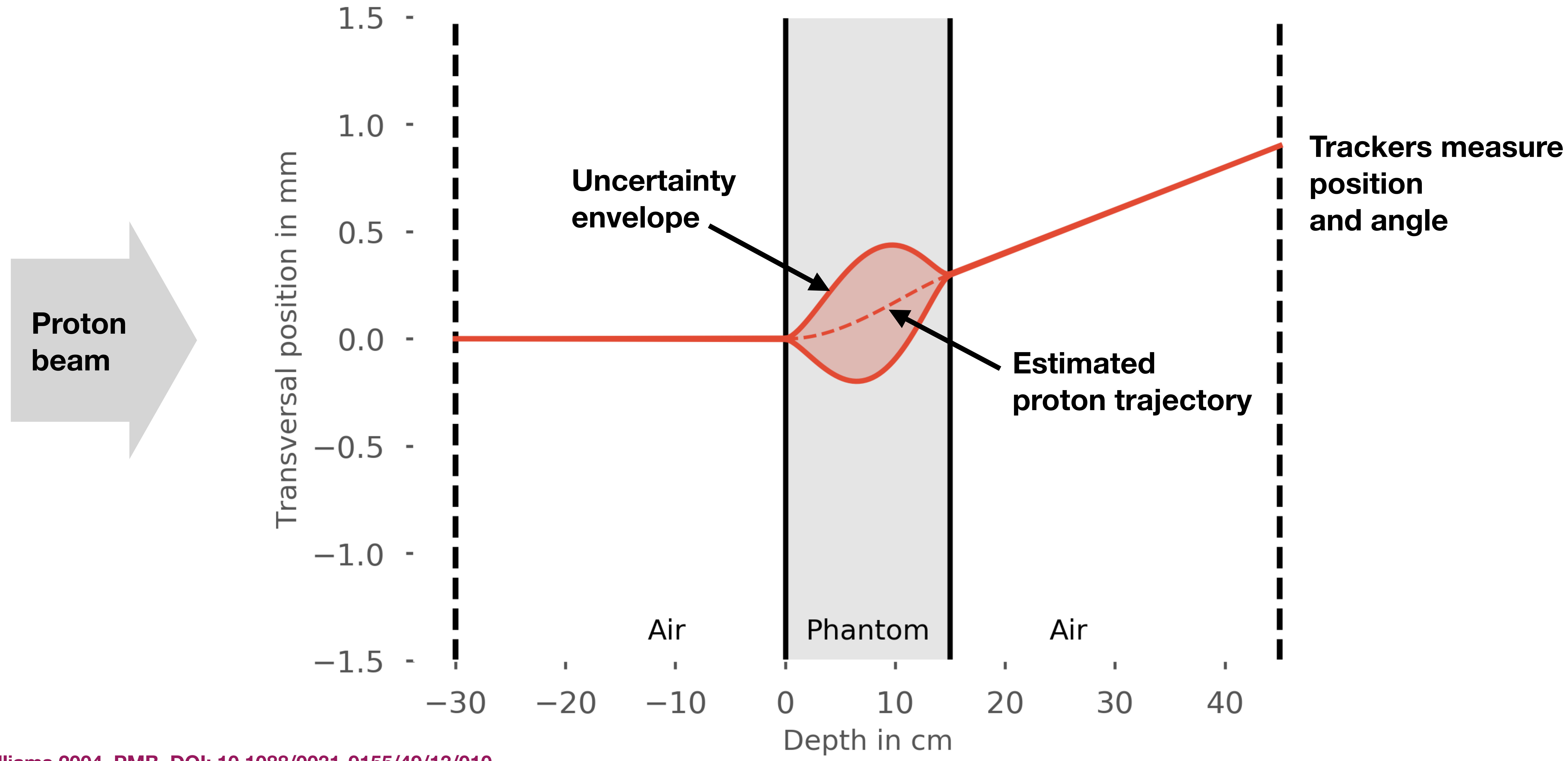
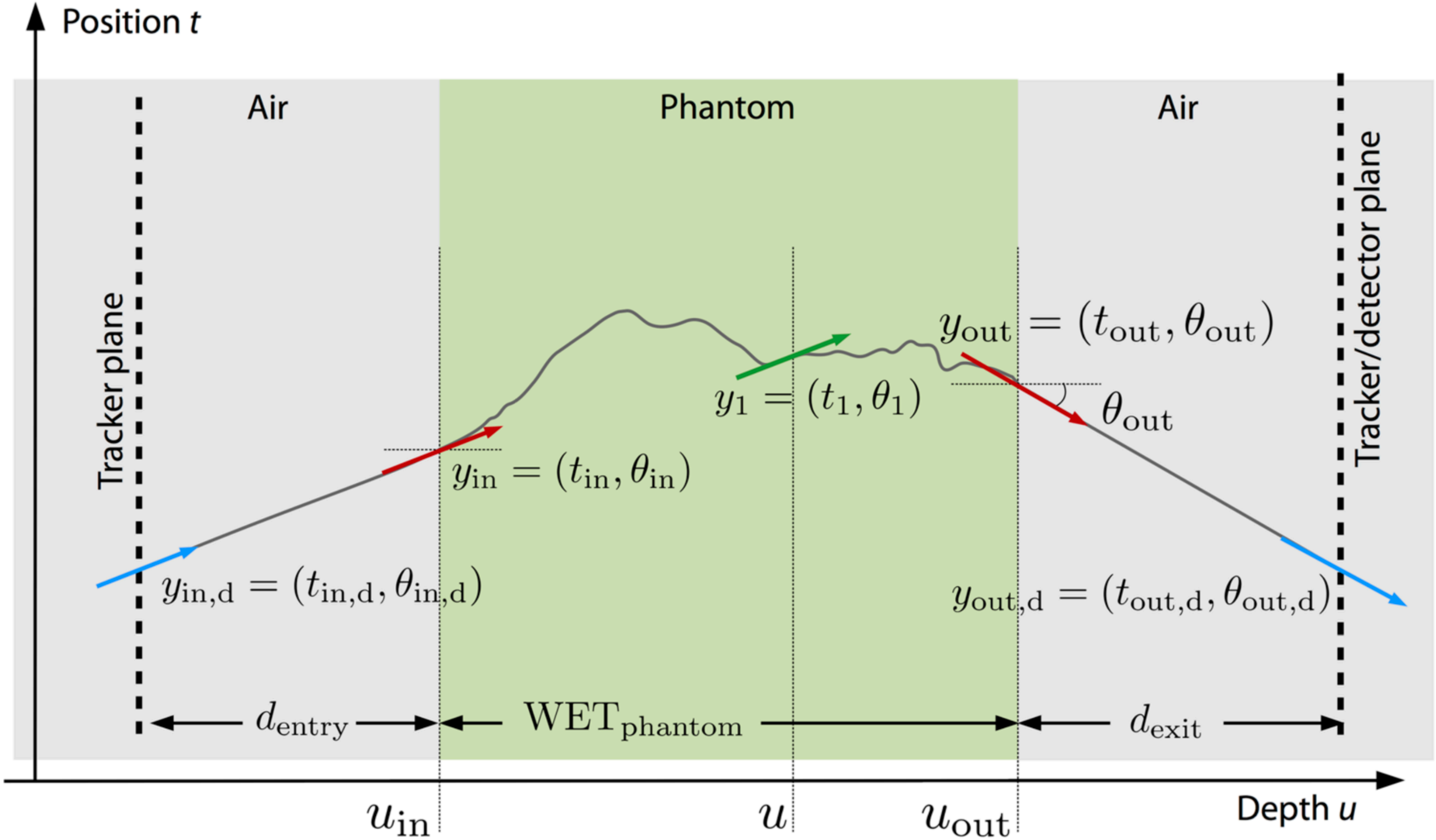
Single tracking set-ups



Williams 2004, PMB, DOI: 10.1088/0031-9155/49/13/010
Schulte 2008, Med. Phys., DOI: 10.1118/1.2986139
Collins-Fekete 2017, PMB, DOI: 10.1088/1361-6560/aa58ce



Position t
Air
Phantom
Air
Tracker plane
Tracker/detector plane
$y_{out} = (t_{out}, \theta_{out})$
$y_1 = (t_1, \theta_1)$
θ_{out}
$y_{in} = (t_{in}, \theta_{in})$
$y_{in,d} = (t_{in,d}, \theta_{in,d})$
$y_{out,d} = (t_{out,d}, \theta_{out,d})$
d_{entry}
$WET_{phantom}$
d_{exit}
u_{in}
u
u_{out}
Depth u

$$L(y_1, y_2 = y_{\mathrm{out}} | y_{\mathrm{in}}) = L_{\mathrm{scat}}(y_{\mathrm{in}} \rightarrow y_1) \times L_{\mathrm{scat}}(y_1 \rightarrow y_2 = y_{\mathrm{out}} | y_{\mathrm{in}})$$

$$L_{\mathrm{scat}}(y_{\mathrm{in}} \rightarrow y_1) \propto \exp\left[-\frac{1}{2}(y_1^T - y_{\mathrm{in}}^T R_0^T)\Sigma_1^{-1}(y_1 - R_0 y_{\mathrm{in}})\right]$$

$$L_{\mathrm{scat}}(y_1 \rightarrow y_2 = y_{\mathrm{out}}) \propto \exp\left[-\frac{1}{2}(y_{\mathrm{out}}^T - y_1^T R_1^T)\Sigma_2^{-1}(y_{\mathrm{out}} - R_1 y_1)\right],$$

$$\Sigma_1 = \begin{pmatrix} \sigma_{t_1}^2 & \sigma_{t_1\theta_1}^2 \\ \sigma_{t_1\theta_1}^2 & \sigma_{\theta_1}^2 \end{pmatrix},\ \Sigma_2 = \begin{pmatrix} \sigma_{t_2}^2 & \sigma_{t_2\theta_2}^2 \\ \sigma_{t_2\theta_2}^2 & \sigma_{\theta_2}^2 \end{pmatrix},\ R_0 = \begin{pmatrix} 1 & u - u_{\mathrm{in}} \\ 0 & 1 \end{pmatrix},\ R_1 = \begin{pmatrix} 1 & u_{\mathrm{out}} - u \\ 0 & 1 \end{pmatrix}$$

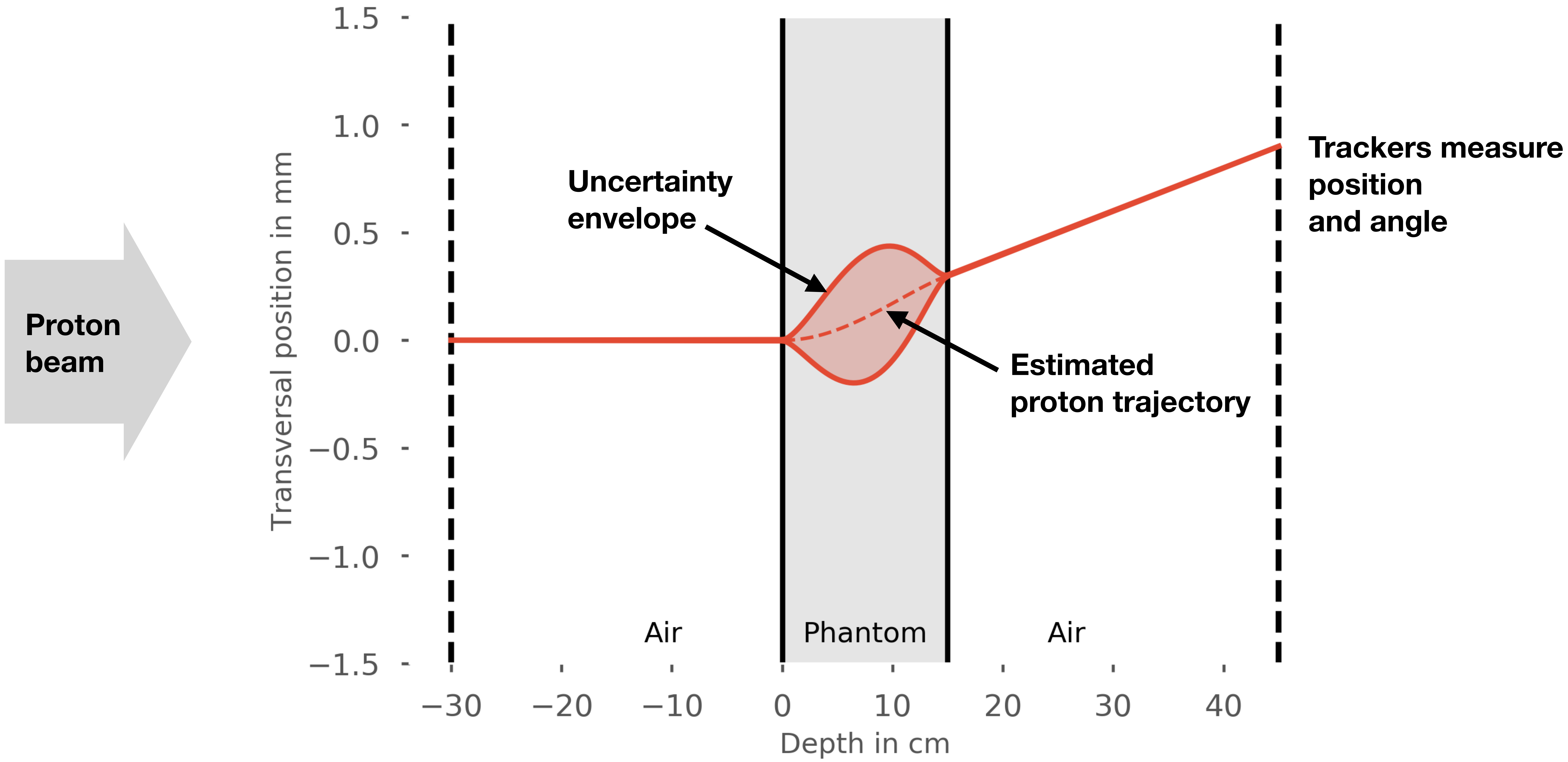
$$\sigma_{t_1}^2 = E_0^2 \left(1 + 0.038 \ln \frac{u - u_{\mathrm{in}}}{X_0}\right)^2 \times \int_{u_{\mathrm{in}}}^{u} \frac{(u - u_{\mathrm{in}})^2}{\beta^2 p^2} \frac{\mathrm{d}u}{X_0}$$

$$L(y_1, y_2 = y_{\text{out}} | y_{\text{in}}) = L_{\text{scat}}(y_{\text{in}} \to y_1) \times L_{\text{scat}}(y_1 \to y_2 = y_{\text{out}} | y_{\text{in}})$$

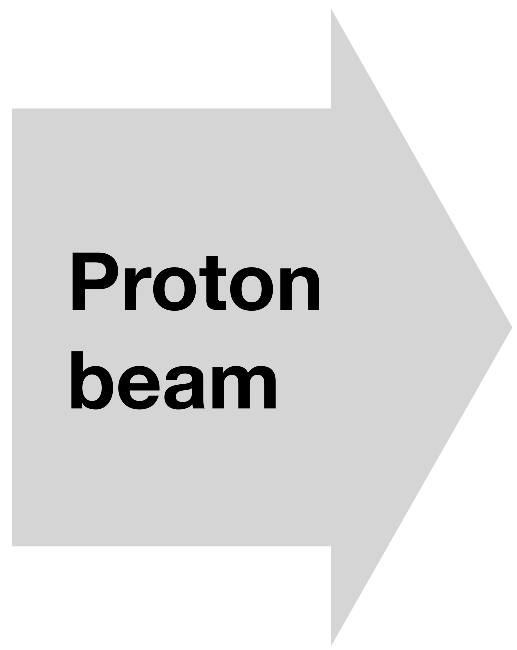
$$\begin{aligned} y_{\text{MLP}}(u) &= \left(\Sigma_1^{-1} + R_1^T \Sigma_2^{-1} R_1\right)^{-1} \cdot \left(\Sigma_1^{-1} R_0 y_{\text{in}} + R_1^T \Sigma_2^{-1} y_{\text{out}}\right) \\ &= R_1^{-1} \Sigma_2 \left(R_1^{-1} \Sigma_2 + \Sigma_1 R_1^T\right)^{-1} \cdot R_0 y_{\text{in}} + \Sigma_1 \left(R_1 \Sigma_1 + \Sigma_2 (R_1^{-1})^T\right)^{-1} \cdot y_{\text{out}} \end{aligned}$$

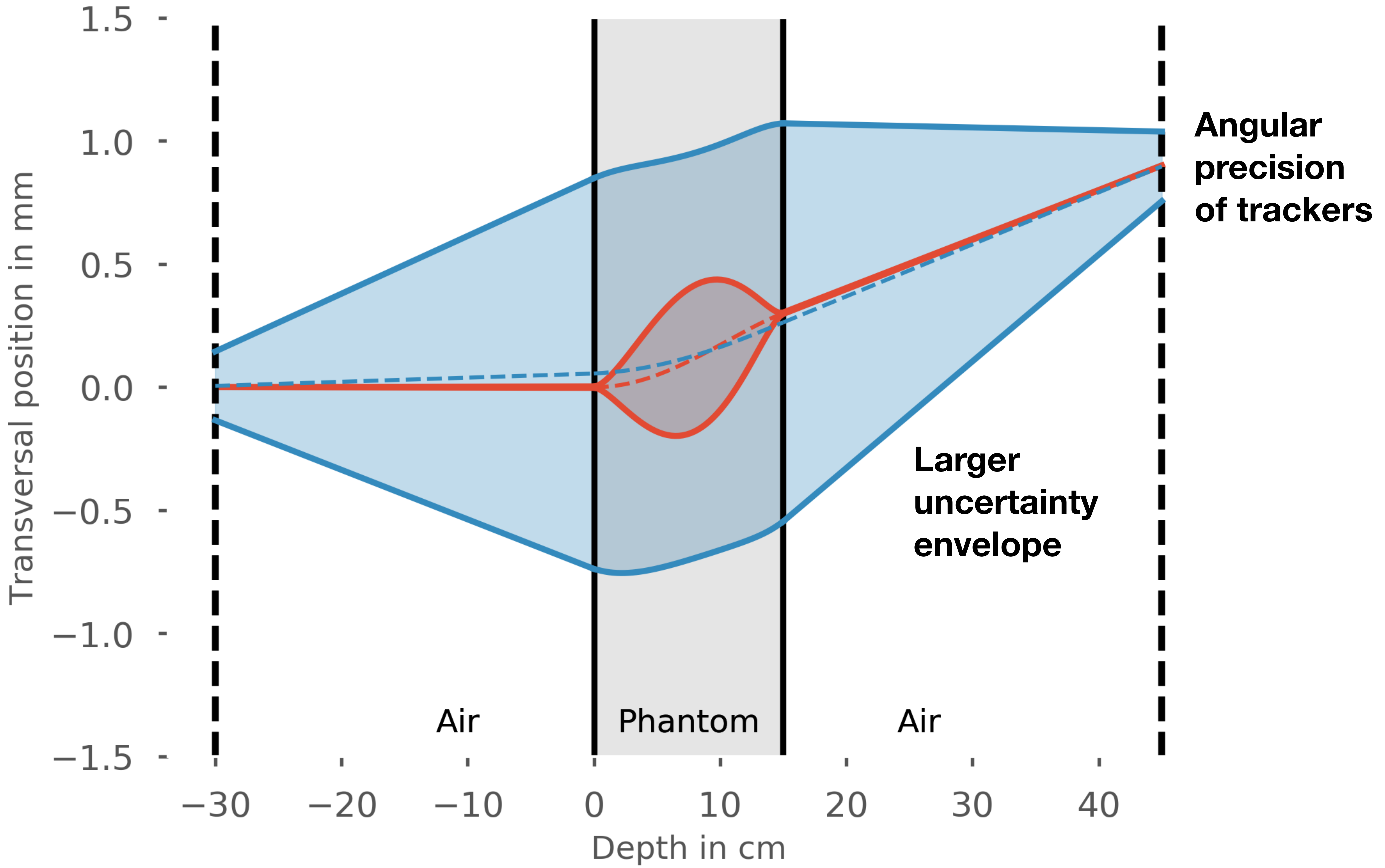
$$\Sigma_{\text{MLP}}(u) = \left(\Sigma_1^{-1} + R_1^T \Sigma_2^{-1} R_1\right)^{-1} = \Sigma_1 \left(\Sigma_2 (R_1^{-1})^T + R_1 \Sigma_1\right)^{-1} \Sigma_2 (R_1^{-1})^T$$

Single tracking set-ups



Tracker uncertainties





Bopp et al. 2014, PMB, DOI: 10.1088/0031-9155/59/23/N197
Penfold S 2011, Radiat. Meas. 46

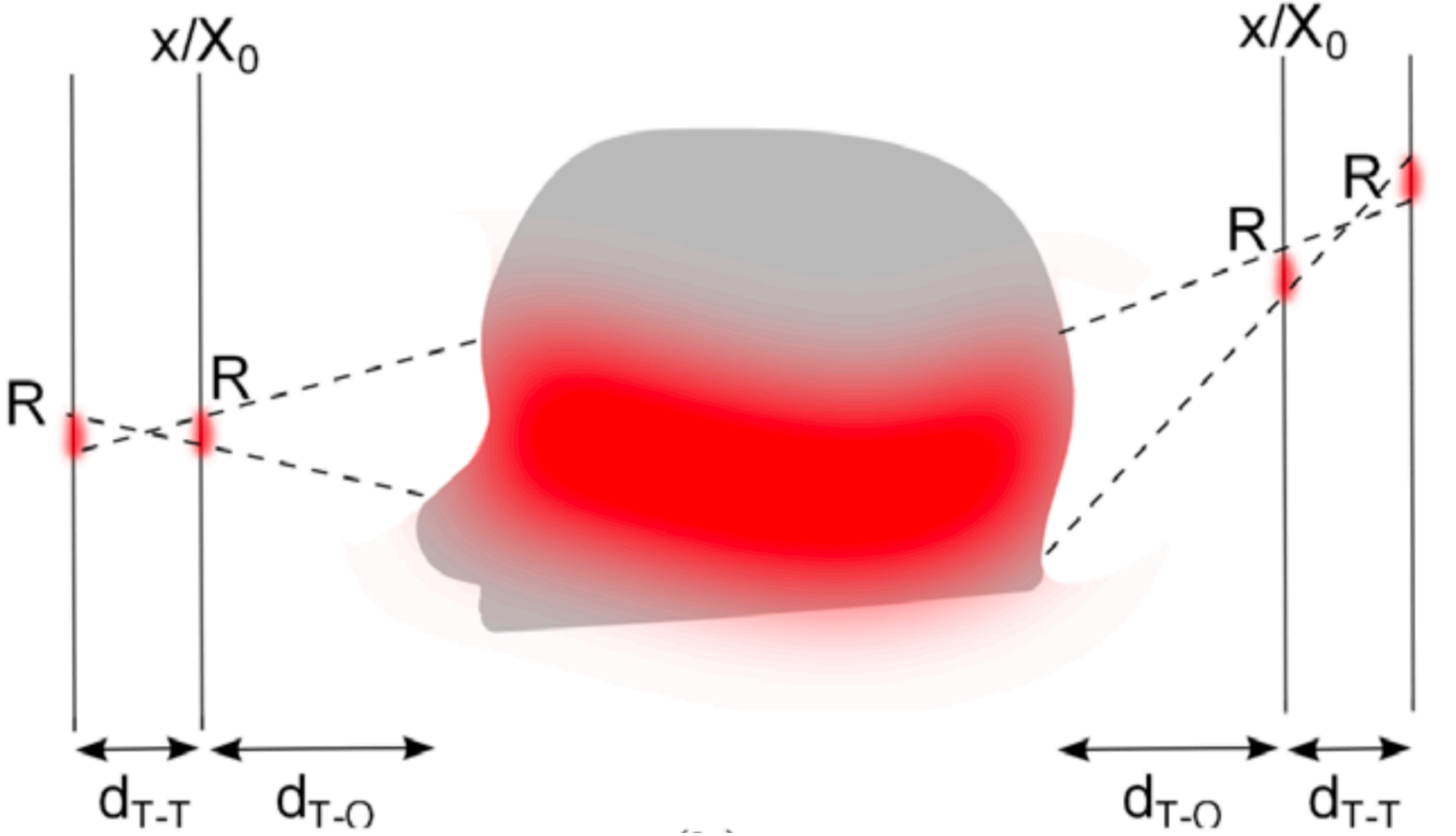


Figure from Bopp et al. 2014, PMB, DOI: 10.1088/0031-9155/59/23/N197

$$\Rightarrow L_{\text{meas}}(\tilde{y}_{\text{in}}, y_{\text{in}}) \propto \exp\left[-\frac{1}{2}(\tilde{y}_{\text{in}} - y_{\text{in}})^T \left(S_{\text{in}}\Sigma_{\text{in}}S_{\text{in}}^T\right)^{-1} (\tilde{y}_{\text{in}} - y_{\text{in}})\right]$$

$$\Rightarrow L_{\text{meas}}(\tilde{y}_{\text{out}}, y_{\text{out}}) \propto \exp\left[-\frac{1}{2}(\tilde{y}_{\text{out}} - y_{\text{out}})^T S_{\text{out}}^T \Sigma_{\text{out}}^{-1} S_{\text{out}}(\tilde{y}_{\text{out}} - y_{\text{out}})\right]$$

$$\Sigma_{\text{in}} = \sigma_p^2 \; T_{\text{in}} \cdot T_{\text{in}}^T + \Sigma_{\text{sc}} \quad \text{and} \quad \Sigma_{\text{out}} = \sigma_p^2 \; T_{\text{out}} \cdot T_{\text{out}}^T + \Sigma_{\text{sc}}$$

$$\Sigma_{\text{sc}} = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_{\text{sc}}^2 \end{pmatrix} \quad \text{with} \quad \sigma_{\text{sc}} = \frac{13.6 \text{ MeV}}{\beta(E)\, p(E)} \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln\left(\frac{x}{X_0}\right)\right]$$

$$T_{\text{out}} = \begin{pmatrix} 1 & 0 \\ -1/d_{\text{T}} & 1/d_{\text{T}} \end{pmatrix} \qquad T_{\text{in}} = \begin{pmatrix} 0 & 1 \\ -1/d_{\text{T}} & 1/d_{\text{T}} \end{pmatrix}$$

Ideal trackers

$$L(y_1, y_2 = y_{\text{out}} | y_{\text{in}}) = L_{\text{scat}}(y_{\text{in}} \to y_1) \times L_{\text{scat}}(y_1 \to y_2 = y_{\text{out}} | y_{\text{in}})$$

Trackers with uncertainties

$$L(y_1, y_2 = \tilde{y}_{\text{out}} | \tilde{y}_{\text{in}}) = \int L_{\text{meas}}(\tilde{y}_{\text{in}}, y_{\text{in}}) L_{\text{scat}}(y_{\text{in}} \to y_1) \mathrm{d}y_{\text{in}} \times \int L_{\text{scat}}(y_1 \to y_2 = y_{\text{out}}) L_{\text{meas}}(\tilde{y}_{\text{out}}, y_{\text{out}}) \mathrm{d}y_{\text{out}}$$

MLP considering tracker uncertainties

$$y_{\mathrm{MLP}}(u) = C_2\left(C_1 + C_2\right)^{-1} R_0 S_{\mathrm{in}} \cdot \tilde{y}_{\mathrm{in,d}} + C_1\left(C_1 + C_2\right)^{-1} R_1^{-1} S_{\mathrm{out}}^{-1} \cdot \tilde{y}_{\mathrm{out,d}}$$

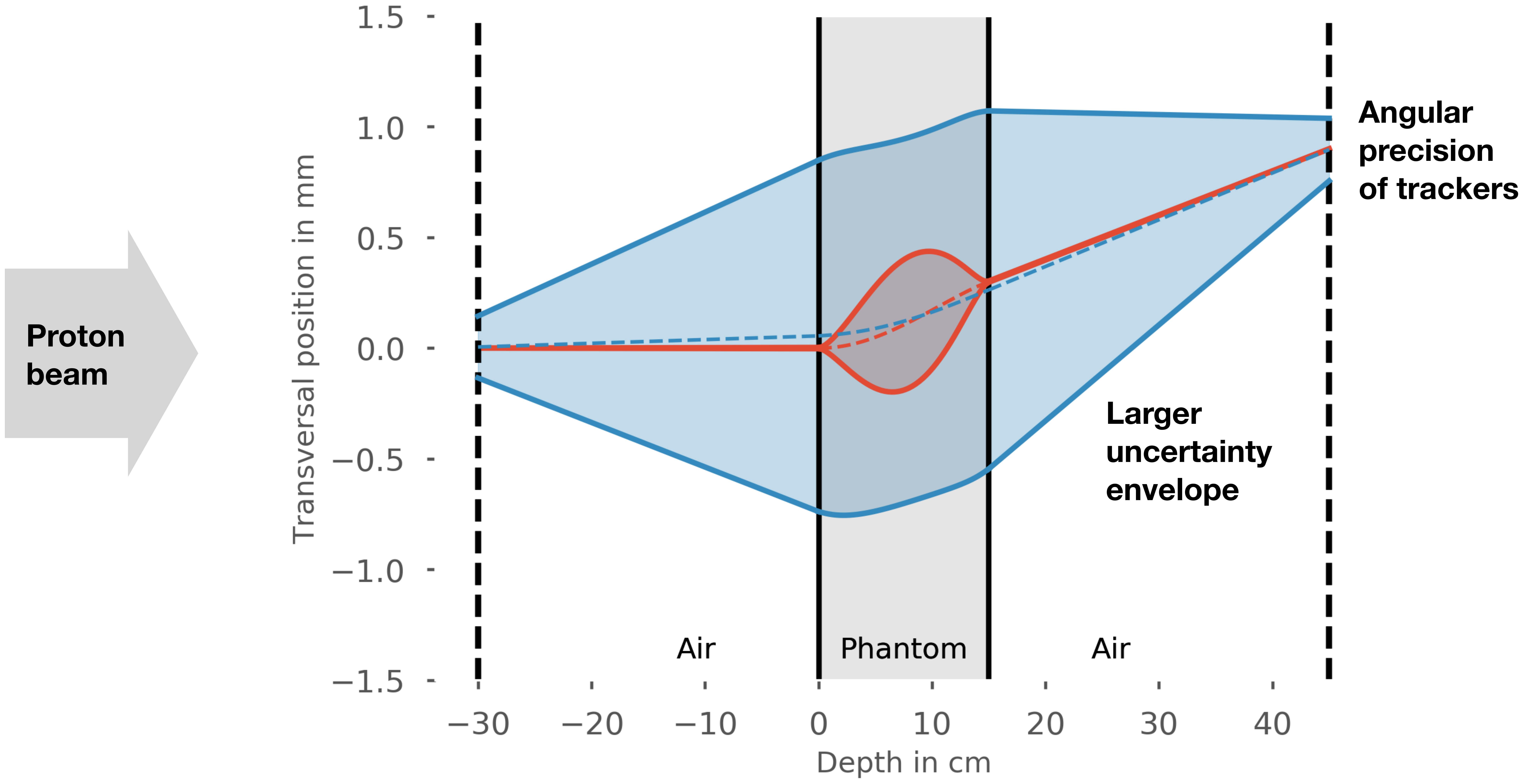
$$\Sigma_{\mathrm{MLP}}(u) = C_1\left(C_1 + C_2\right)^{-1} C_2$$

$$\sigma_{\mathrm{MLP}}(u) = \left(\Sigma_{\mathrm{MLP}}(u)\right)_{1,1}$$

$$C_1 = R_0 S_{\mathrm{in}} \Sigma_{\mathrm{in}} S_{\mathrm{in}}^T R_0^T + \Sigma_1$$

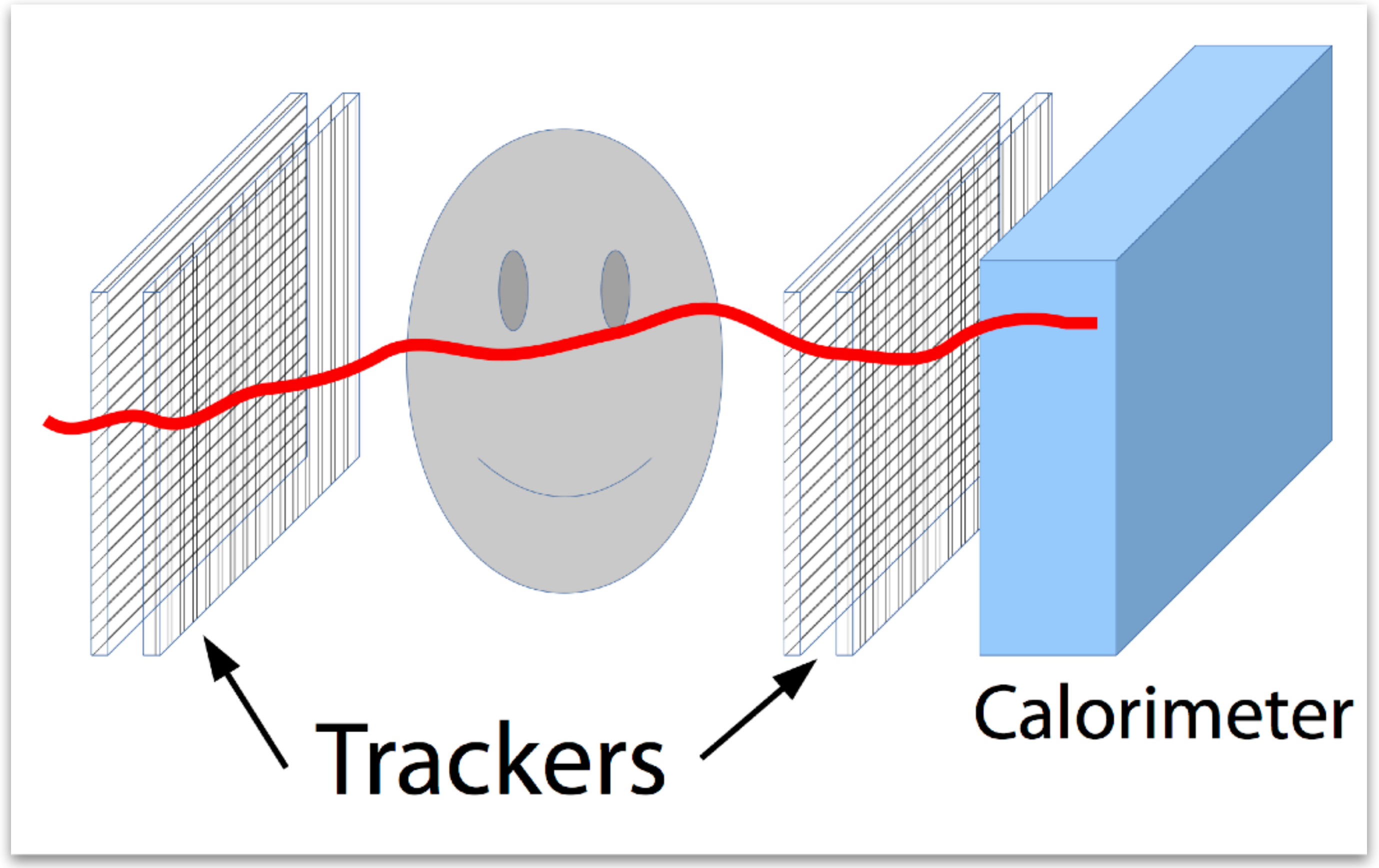
$$C_2 = R_1^{-1} S_{\mathrm{out}}^{-1} \Sigma_{\mathrm{out}} (S_{\mathrm{out}}^{-1})^T (R_1^{-1})^T + R_1^{-1} \Sigma_2 (R_1^{-1})^T$$

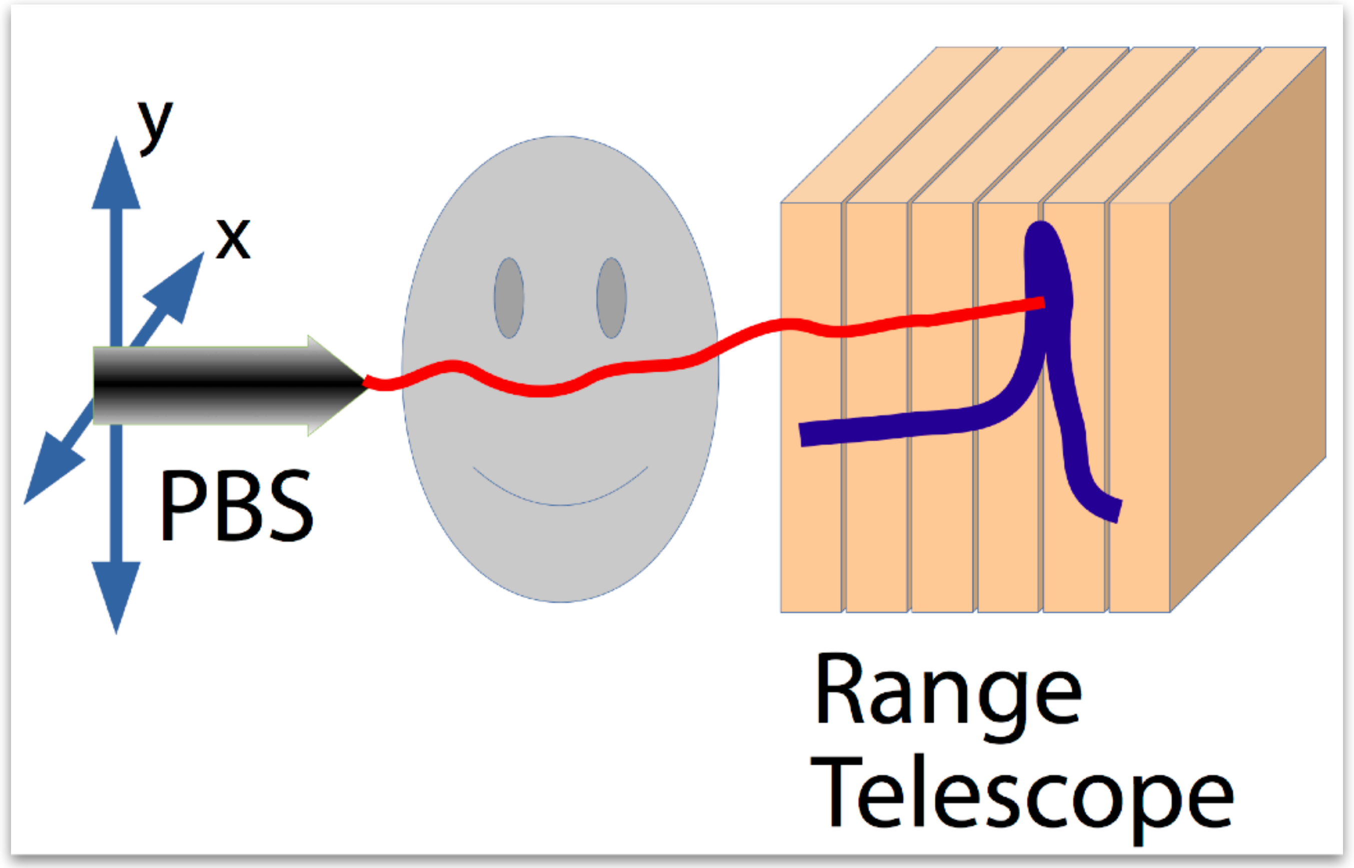
Tracker uncertainties

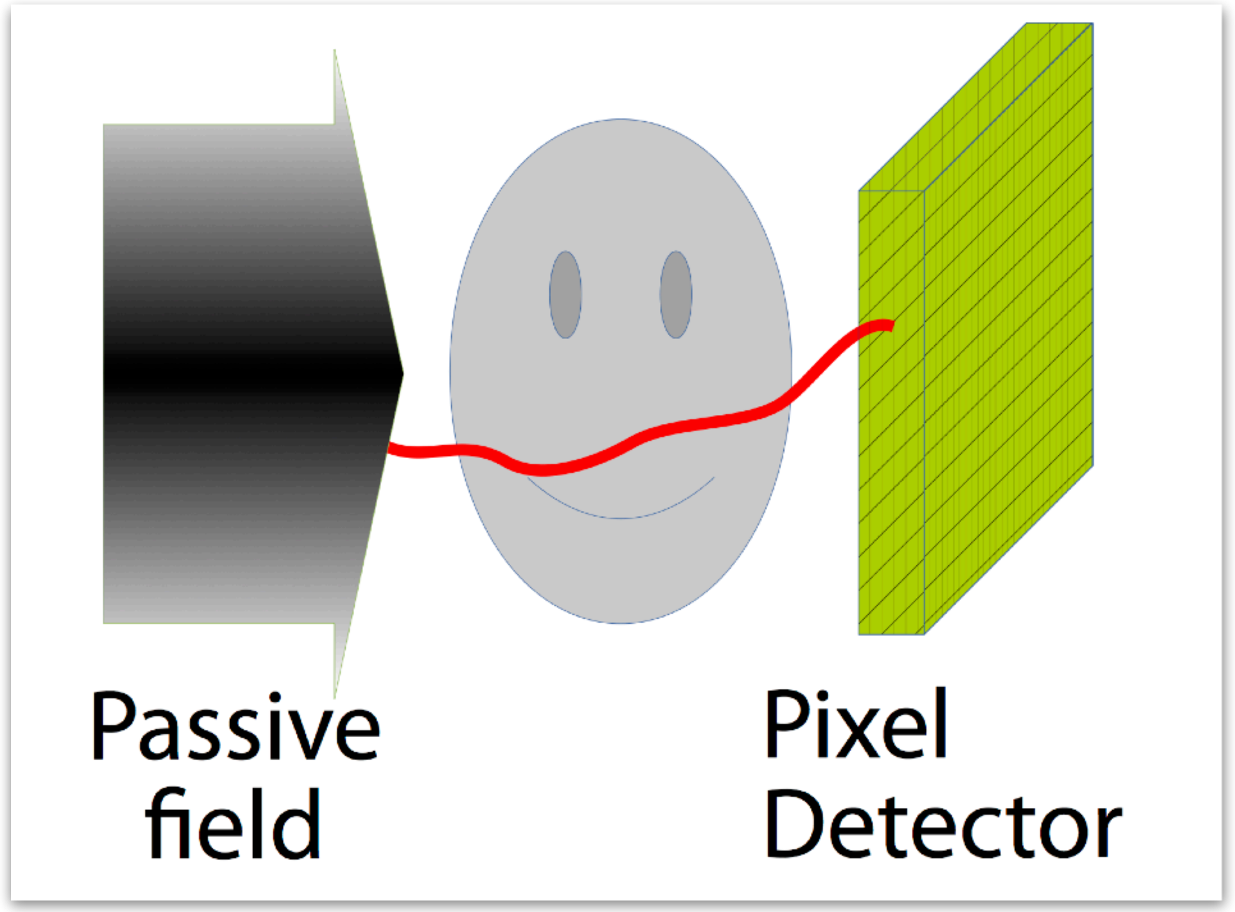


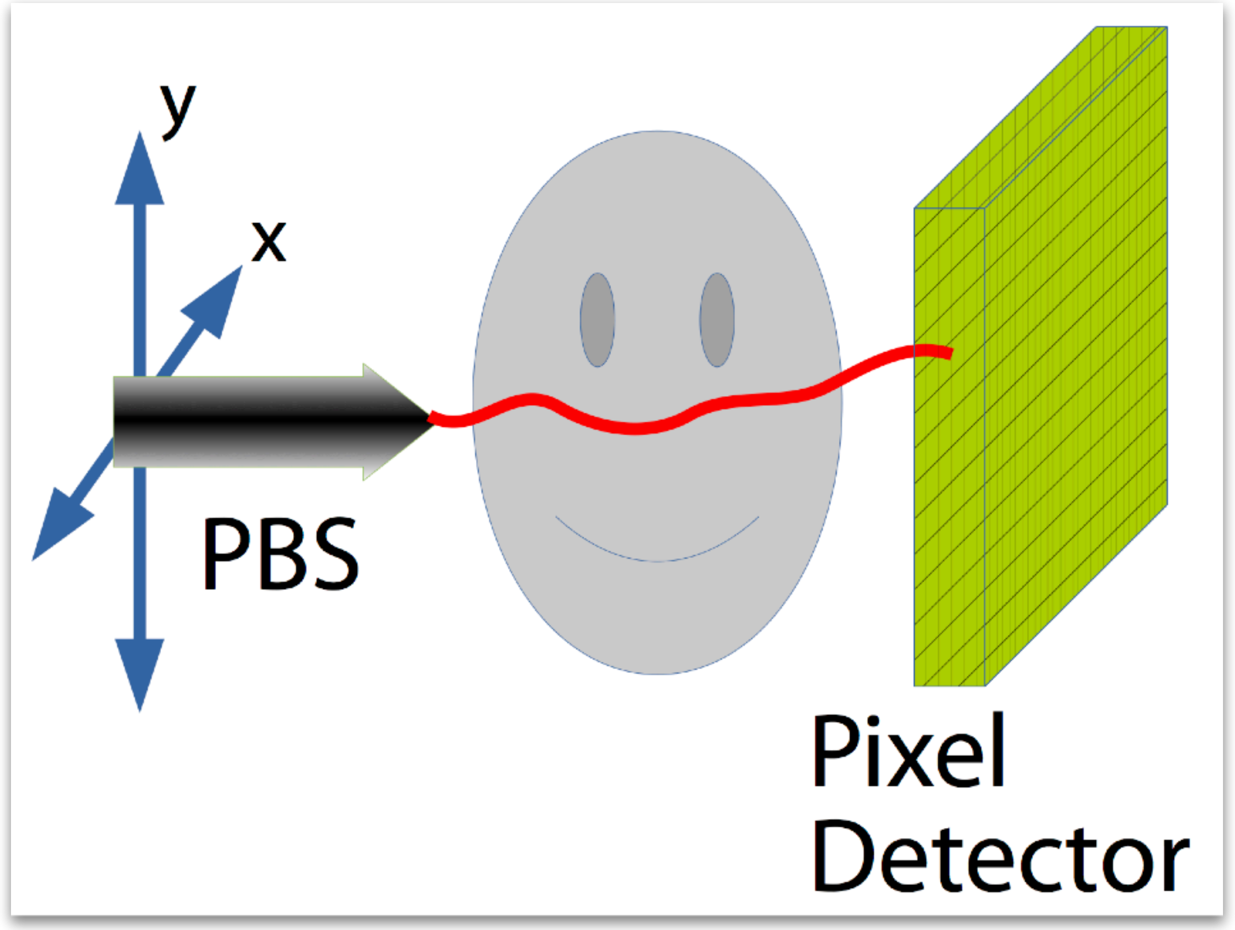
Bopp et al. 2014, PMB, DOI: 10.1088/0031-9155/59/23/N197
Penfold S 2011, Radiat. Meas. 46

Apply formalism to integral mode set-ups

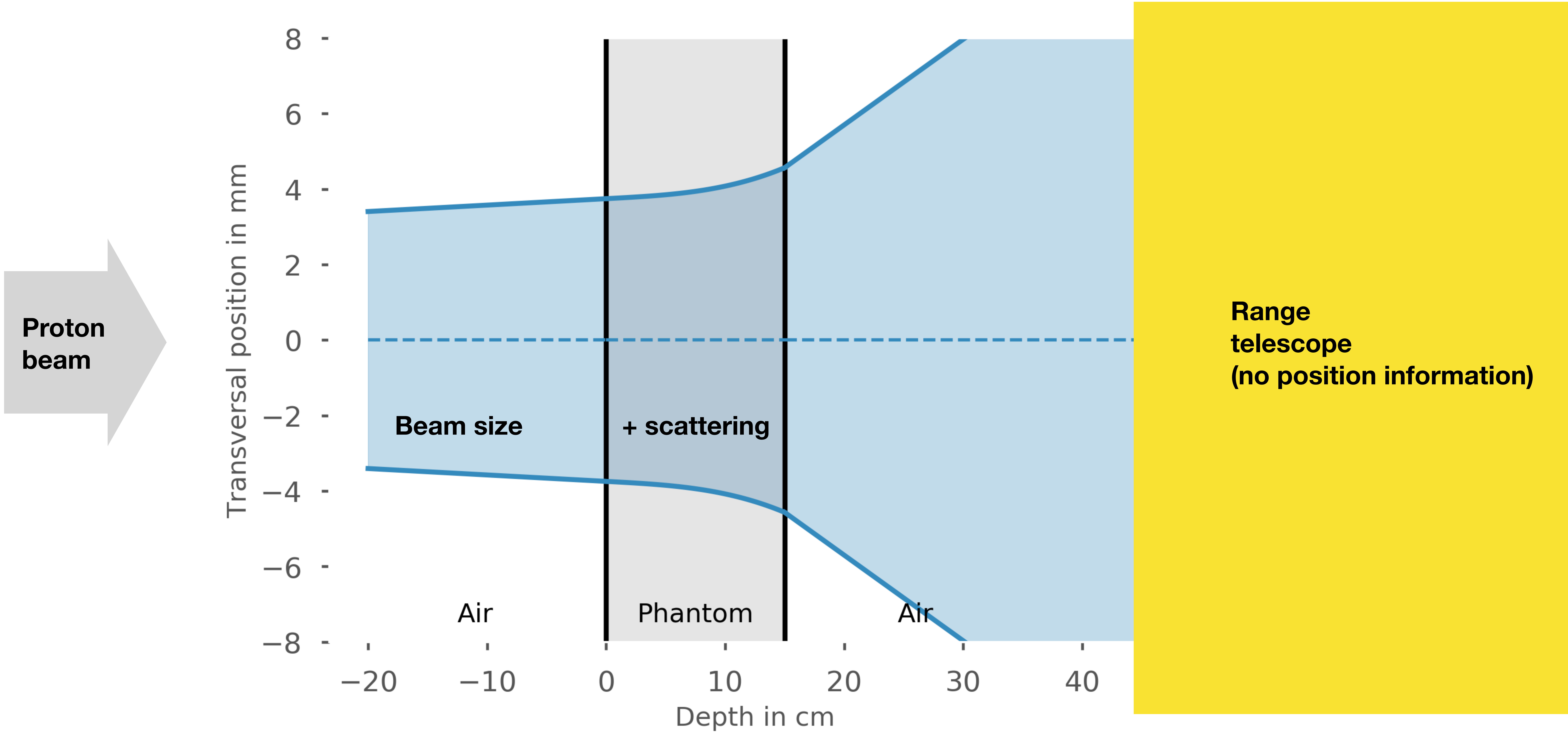




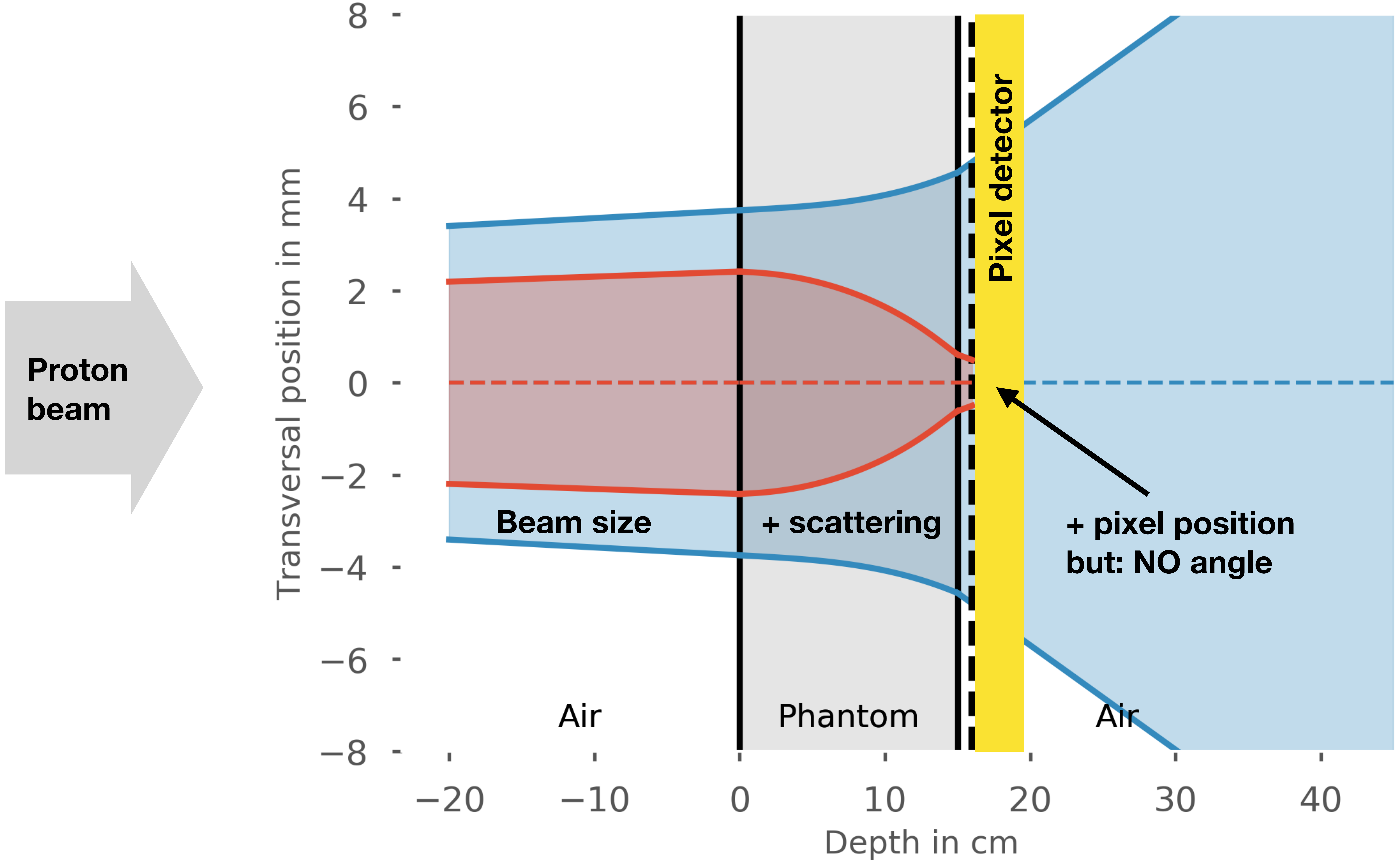




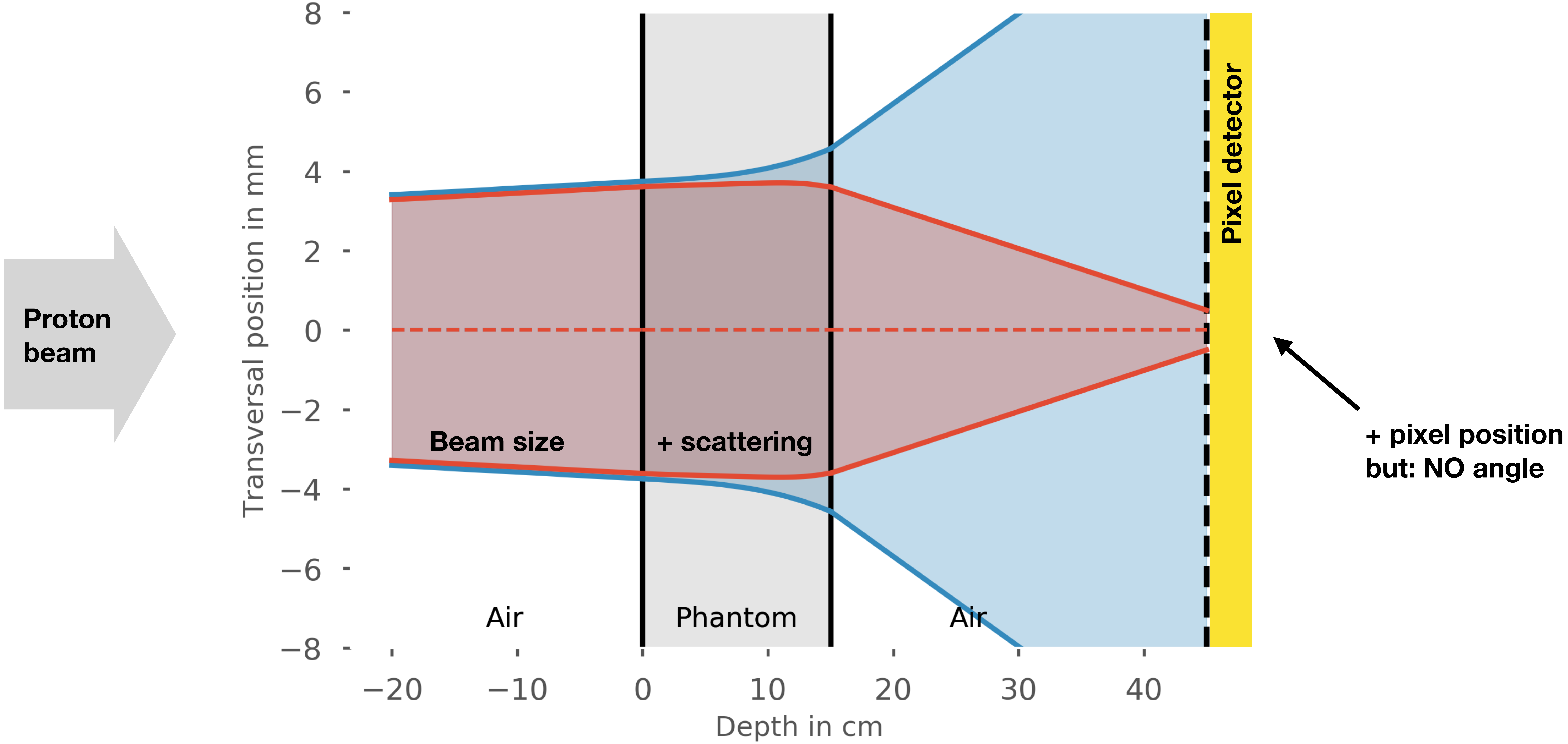
PBS-based set-ups



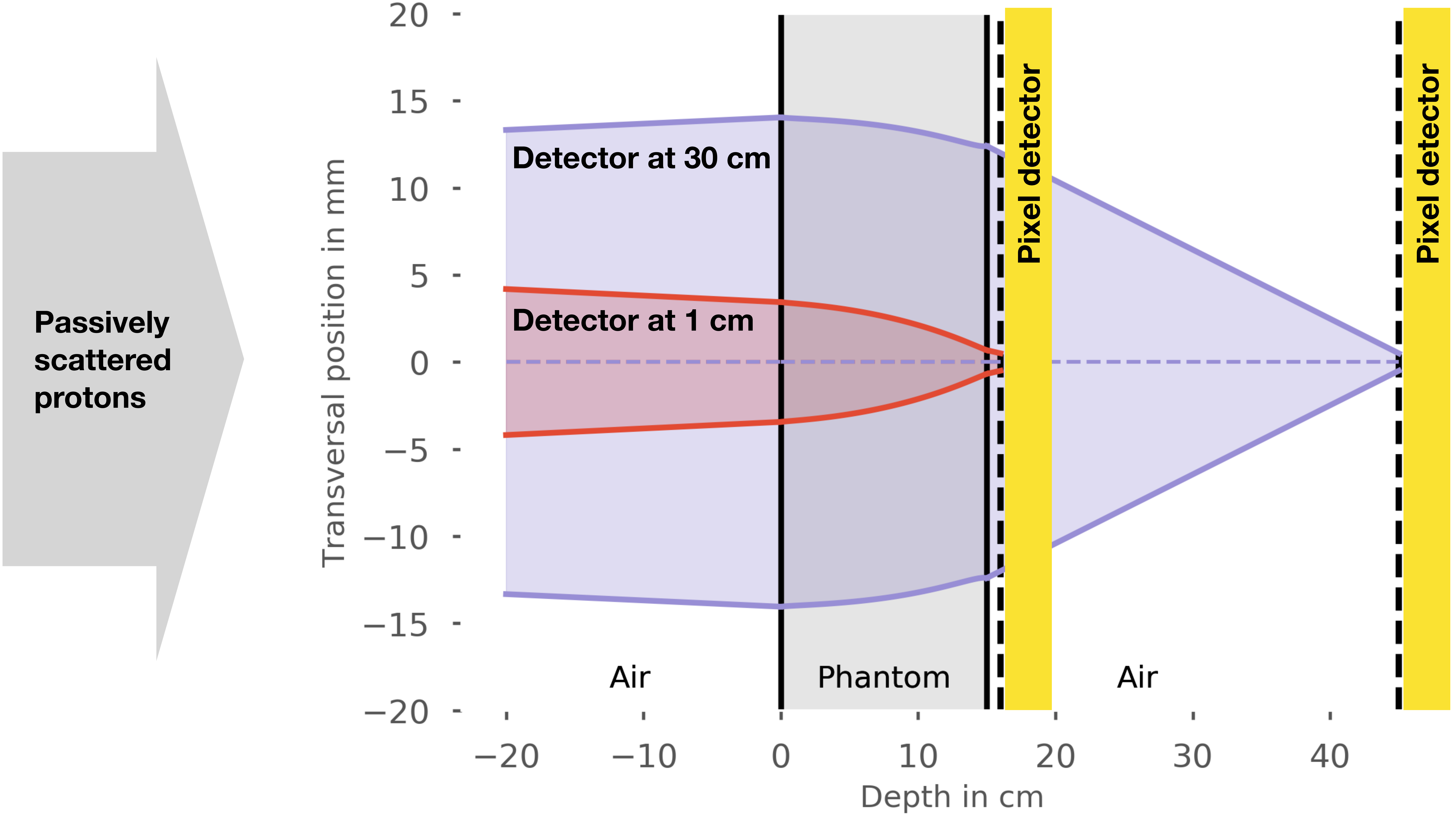
PBS-based set-ups



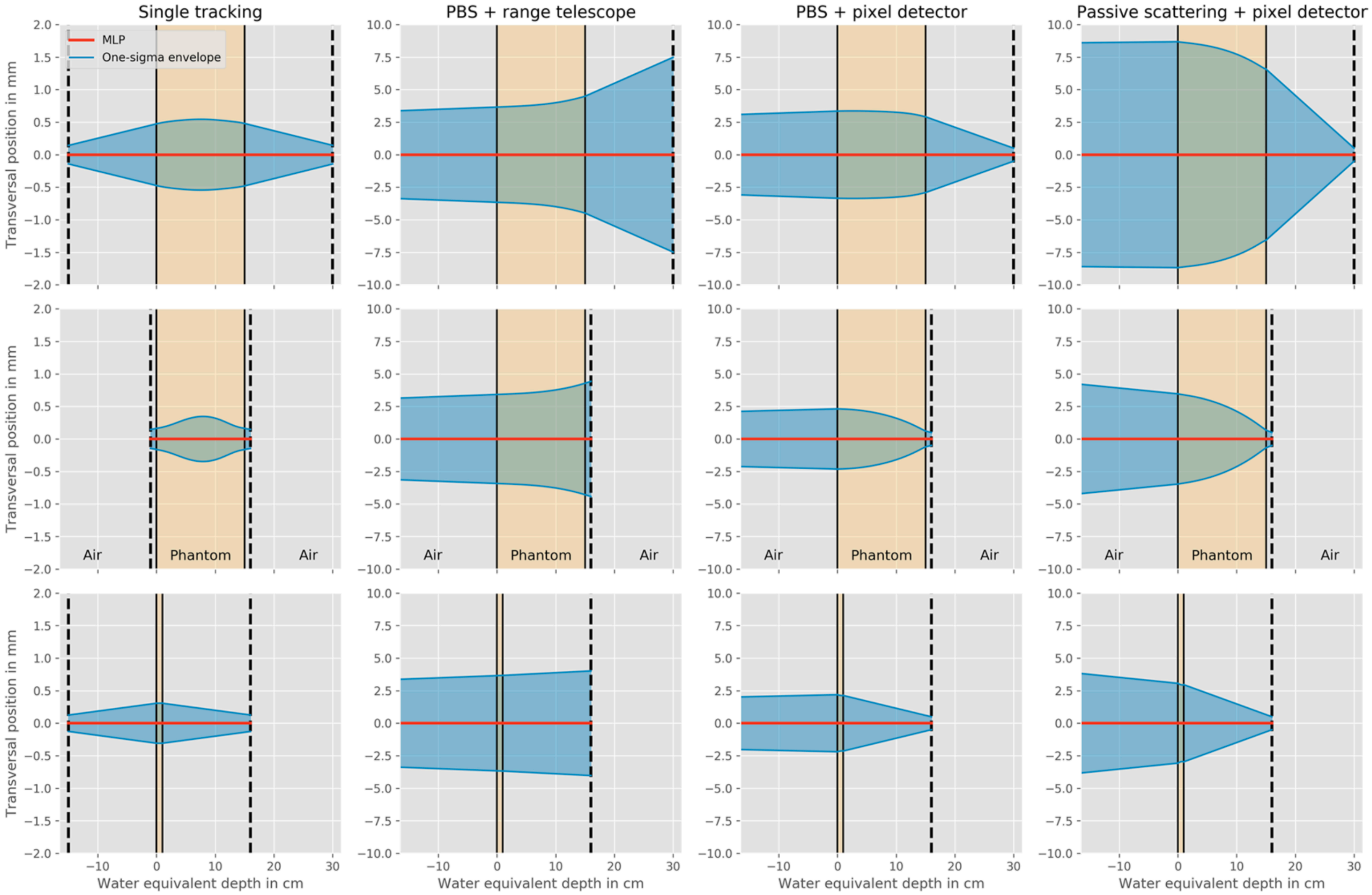
PBS-based set-ups



Passive scattering set-ups

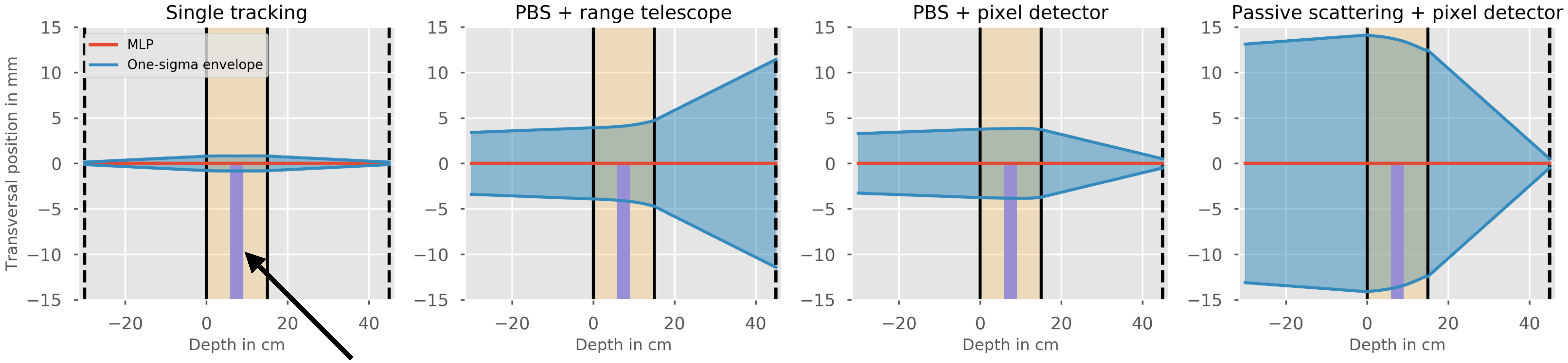


Set-up parameter	$\sigma_{t_{in}}$	$\sigma_{\theta_{in}}$	$\sigma_{t_{out}}$	$\sigma_{\theta_{out}}$
Single tracking idealised	0	0	0	0
PBS + range telescope idealised	0	0	$\rightarrow \infty$	$\rightarrow \infty$
PBS + pixel detector idealised	0	0	0	$\rightarrow \infty$
Passive scattering + pixel detector idealised	$\rightarrow \infty$	0	0	$\rightarrow \infty$
Single tracking (see section 28)	0.15 mm	3 mrad	0.15 mm	3 mrad
Single tracking without angle measurement	0.5 mm	15 mrad	0.5 mm	$\rightarrow \infty$ (45°)
PBS + range telescope	8/2.35 mm	0.1 mrad	20 cm	$\rightarrow \infty$ (45°)
passive scattering + pixel detector	20 cm	15 mrad	0.5 mm	$\rightarrow \infty$ (45°)
PBS + pixel detector	8/2.35 mm	0.1 mrad	0.5 mm	$\rightarrow \infty$ (45°)



Single tracking
PBS + range telescope
PBS + pixel detector
Passive scattering + pixel detector
MLP
One-sigma envelope
Transversal position in mm
Air
Phantom
Air
Water equivalent depth in cm

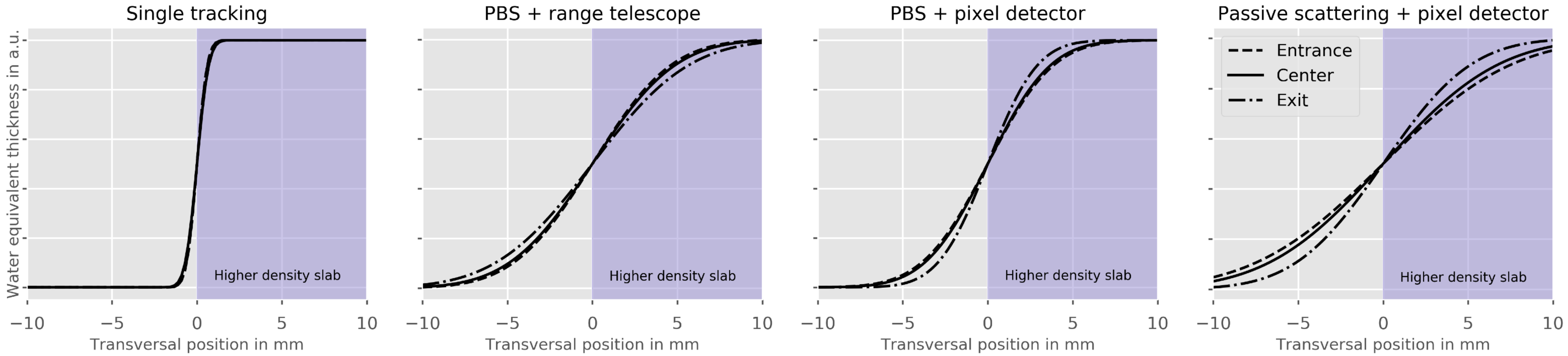
Measure of spatial resolution



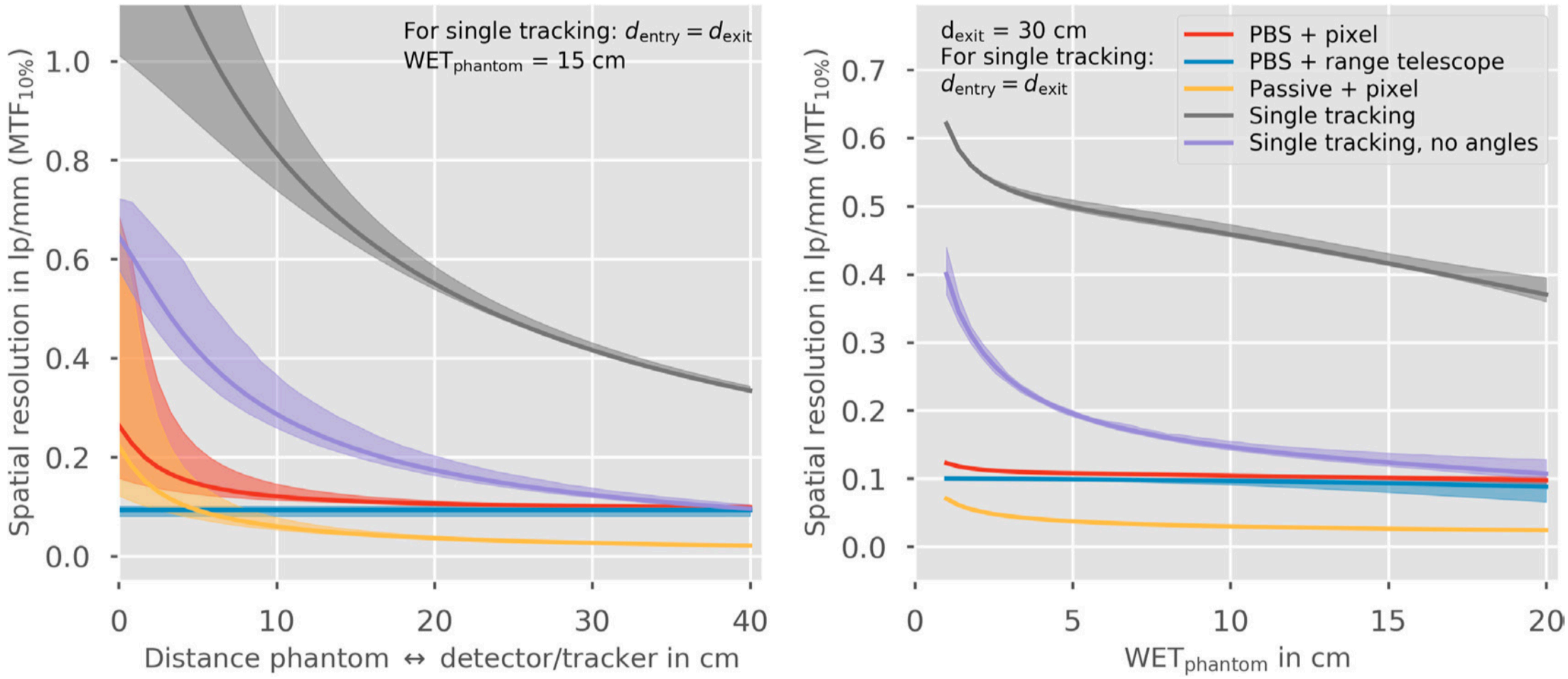
Higher density slab

Edge spread distribution

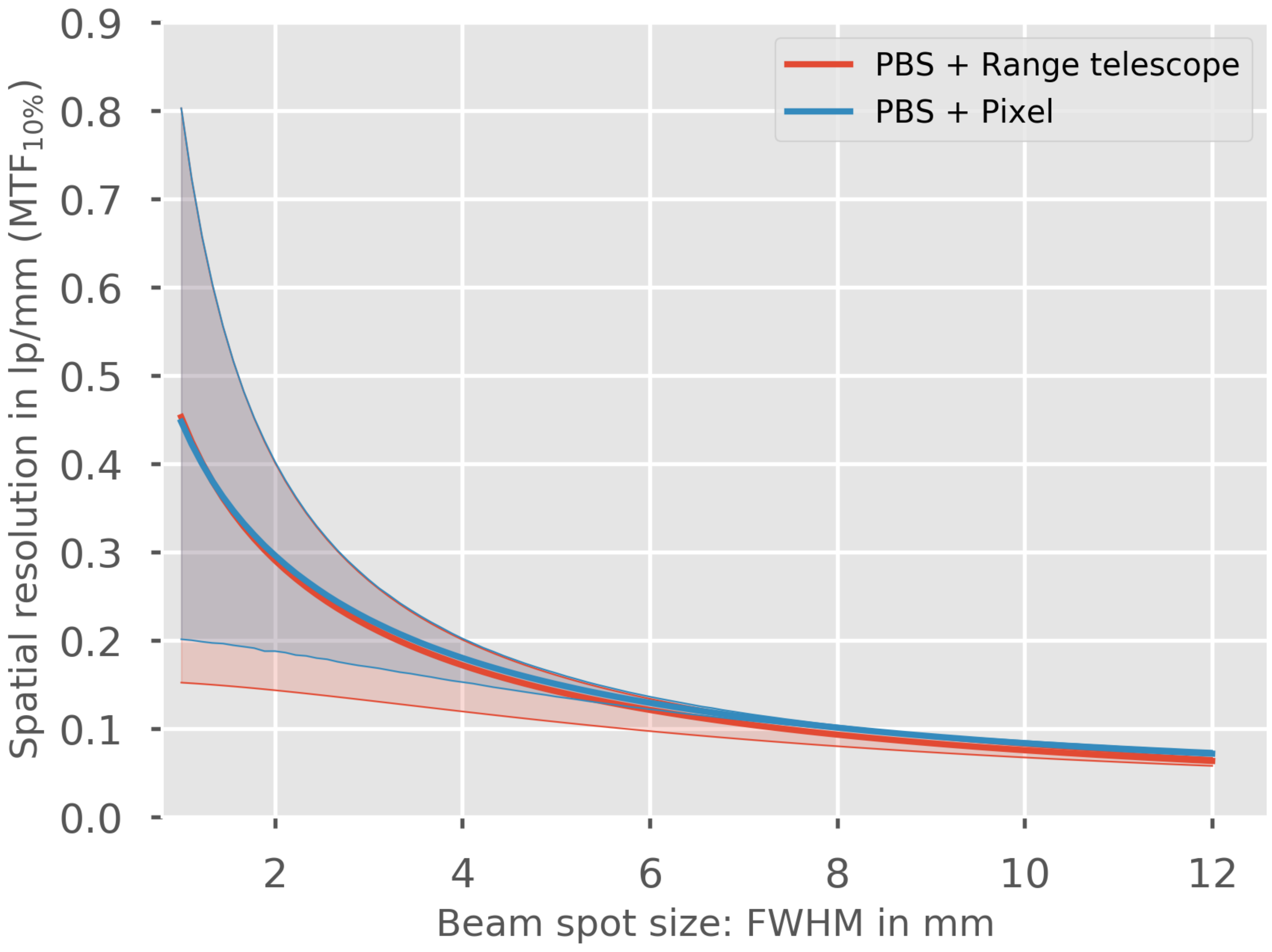
Spatial resolution in lp/mm



Impact of detector distance on spatial resolution

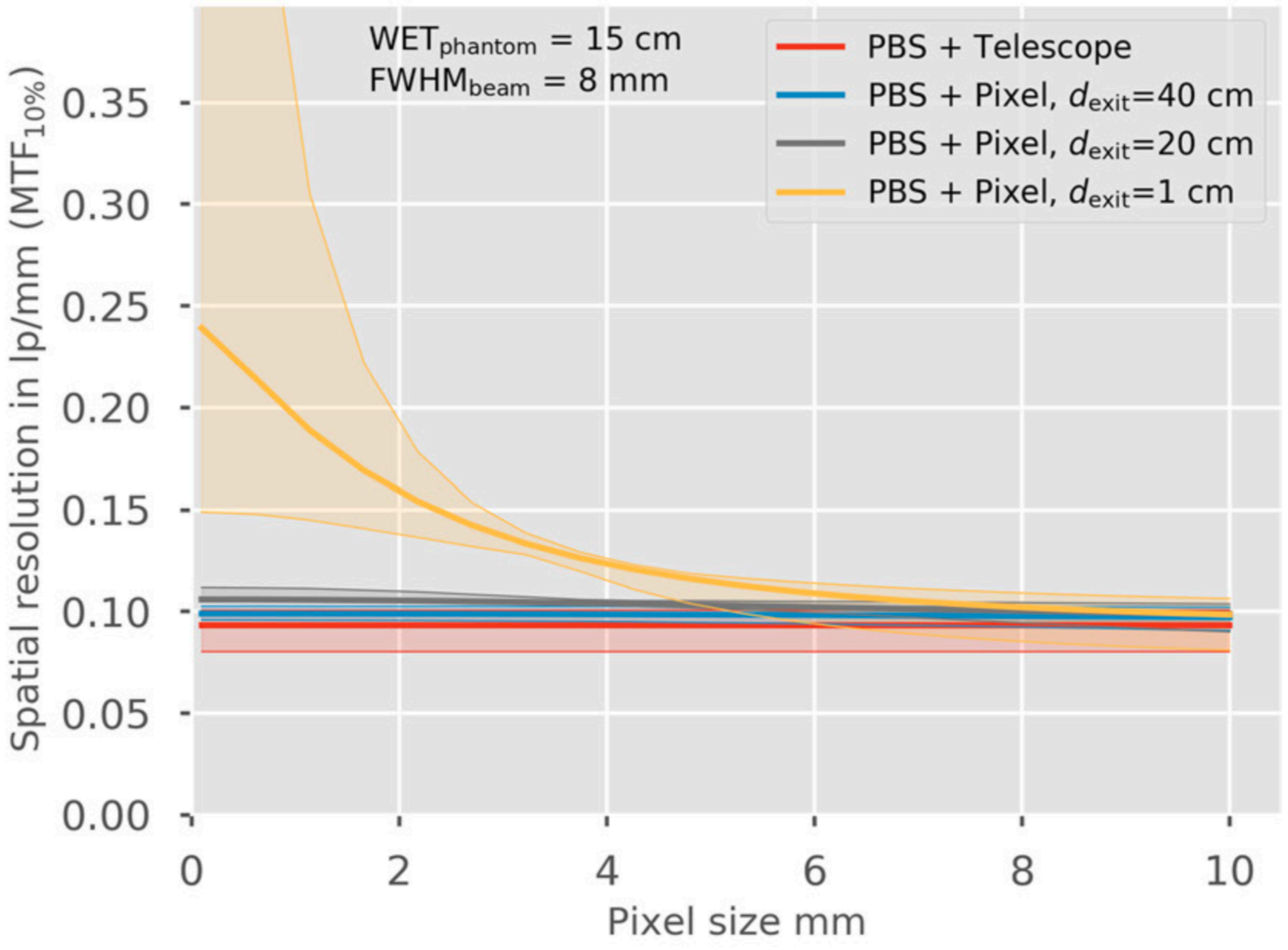


Impact of beam size in PBS set-ups



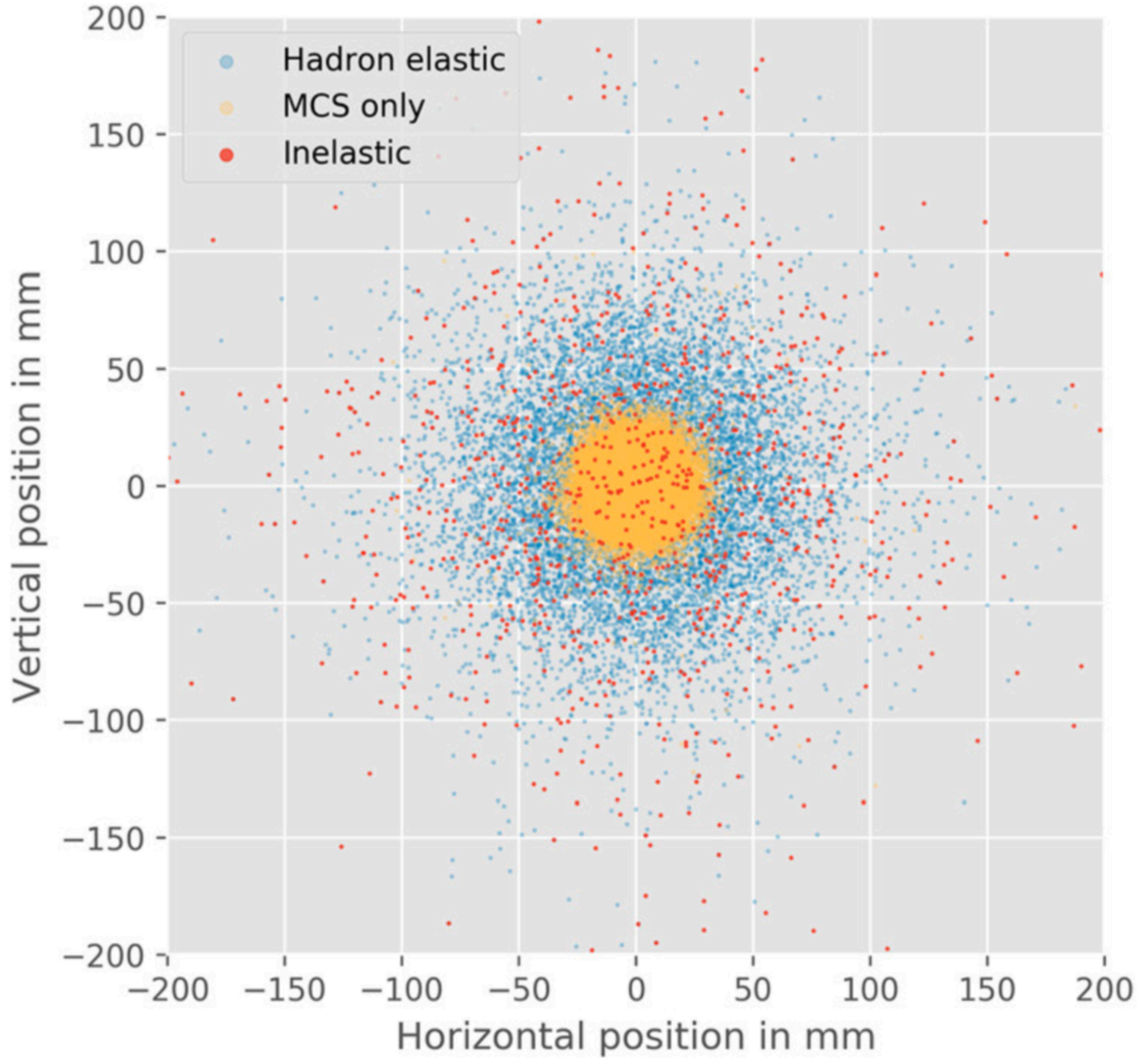
Small beam size greatly improves spatial resolution

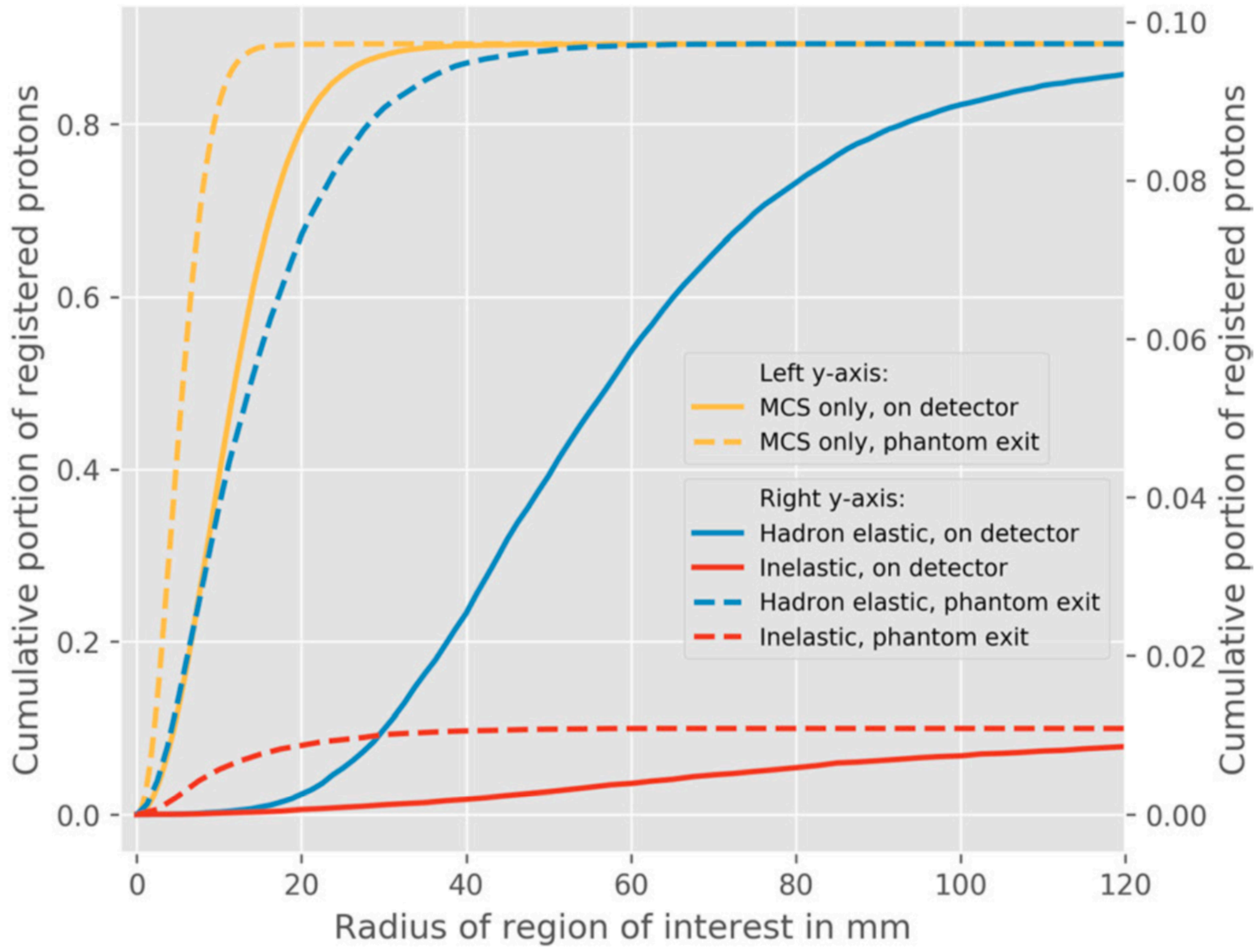
Pixel size



… does not really matter unless the detector is placed very close yo the phantom

So pixels are useless?



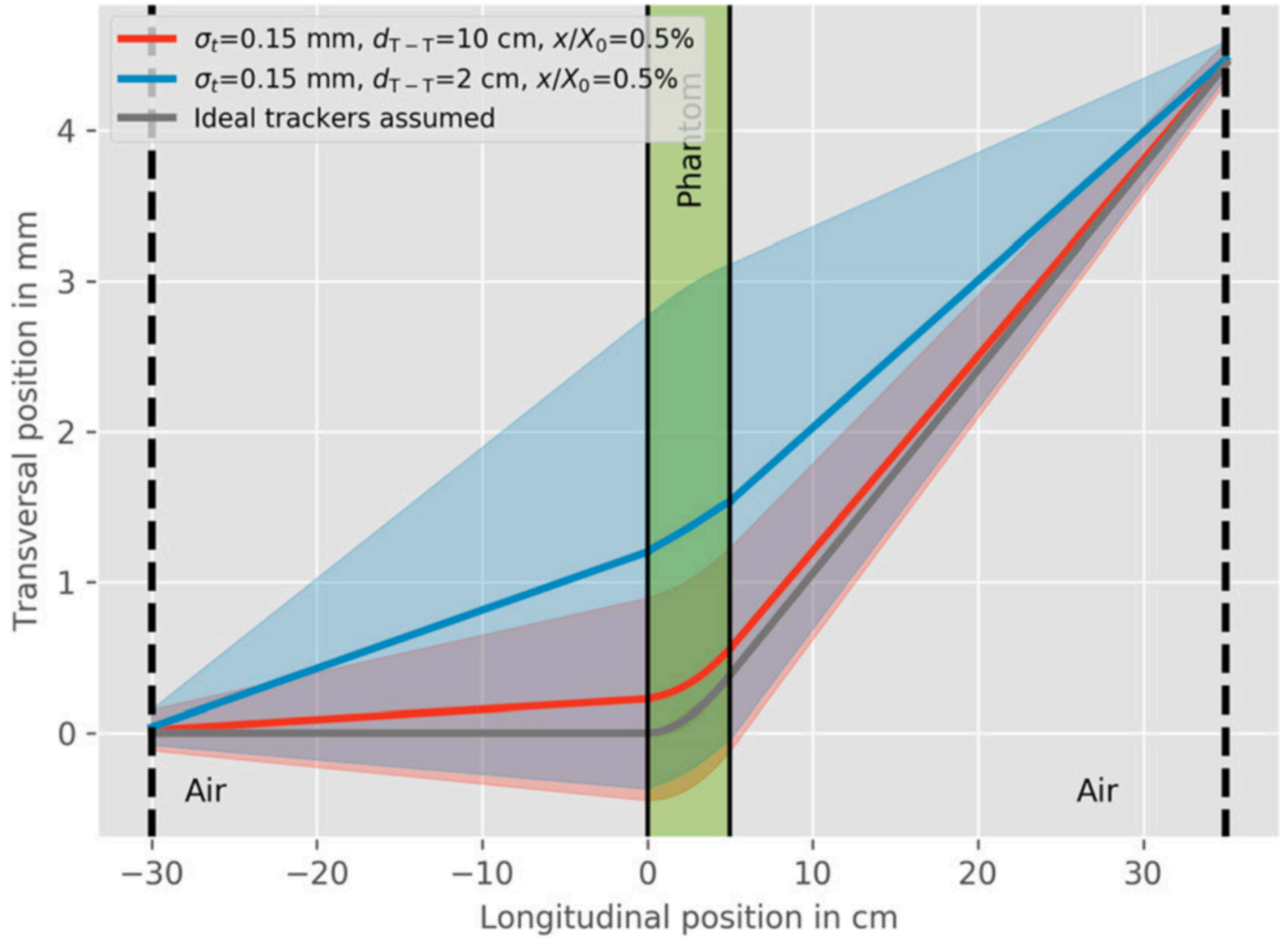


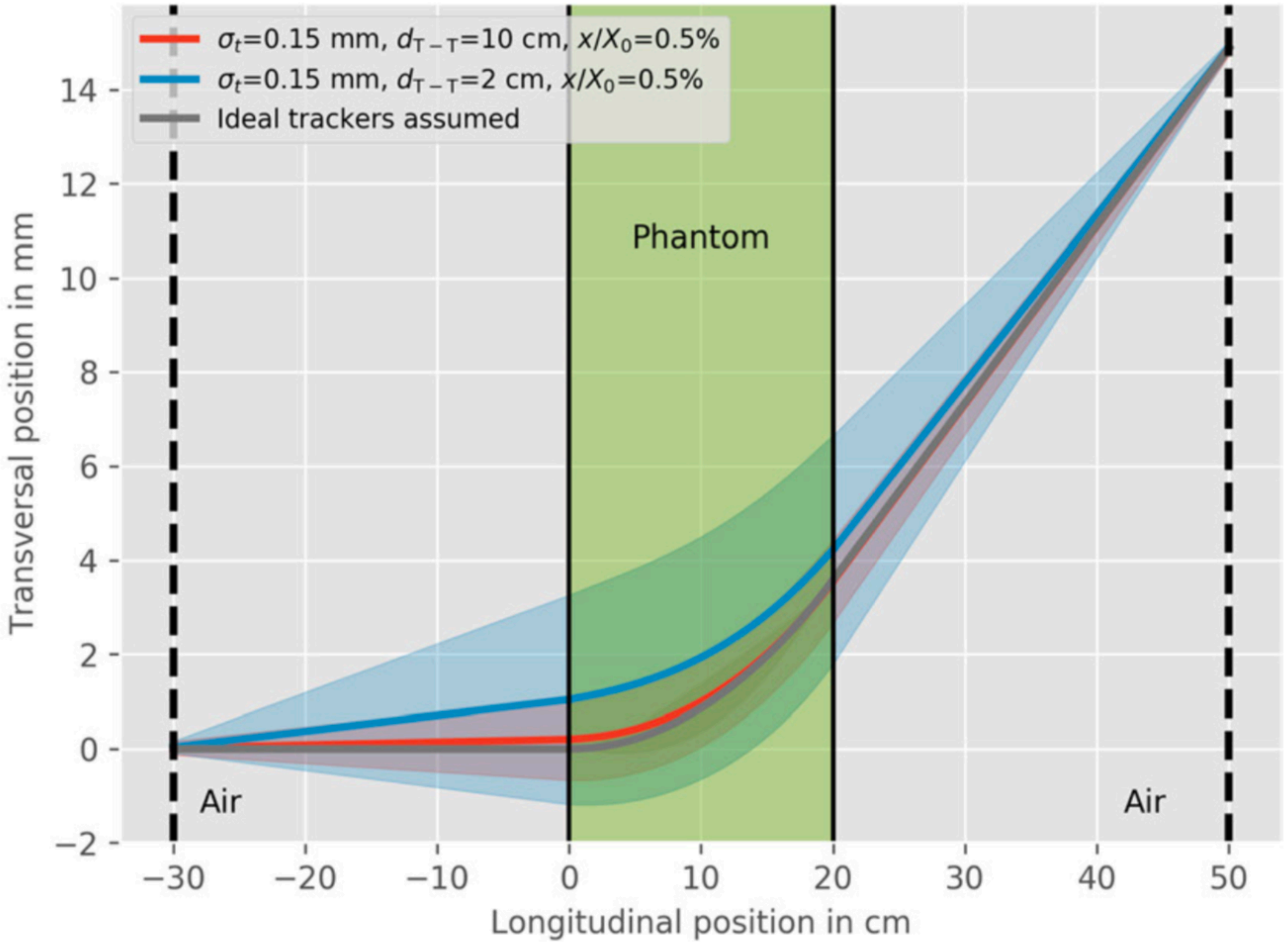
Impact of experimental uncertainties on MLP

$$y_{\mathrm{MLP}}(u) = C_2 \left(C_1 + C_2\right)^{-1} R_0 S_{\mathrm{in}} \cdot \tilde{y}_{\mathrm{in,d}} + C_1 \left(C_1 + C_2\right)^{-1} R_1^{-1} S_{\mathrm{out}}^{-1} \cdot \tilde{y}_{\mathrm{out,d}}$$

$$C_1 = R_0 S_{\mathrm{in}} \Sigma_{\mathrm{in}} S_{\mathrm{in}}^T R_0^T + \Sigma_1$$

$$C_2 = R_1^{-1} S_{\mathrm{out}}^{-1} \Sigma_{\mathrm{out}} (S_{\mathrm{out}}^{-1})^T (R_1^{-1})^T + R_1^{-1} \Sigma_2 (R_1^{-1})^T$$

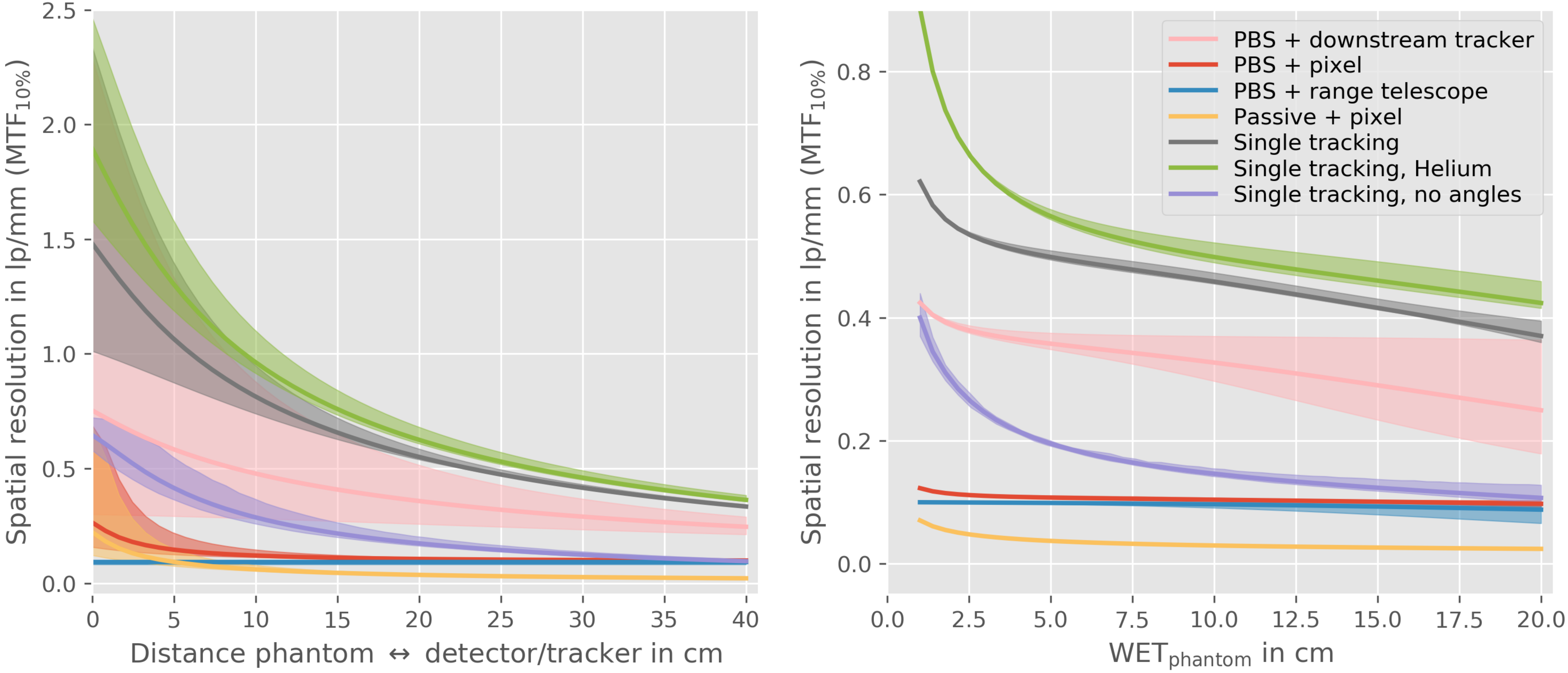




Conclusion

- List mode:
  tracker uncertainty very important
  spatial resolution degrades drastically with tracker distance
- Passive field: Must put detector close to phantom/patient
- PBS: Small beam size improves spatial resolution a lot
- Pixel size is irrelevant for spatial resolution
  but pixels allows for region of interest filtering
- Refine MLP estimation?

Plotted yesterday night after a couple of beers*



***interpret with care**

Merci !

Open Post-Doc position
on proton CT reconstruction in our group