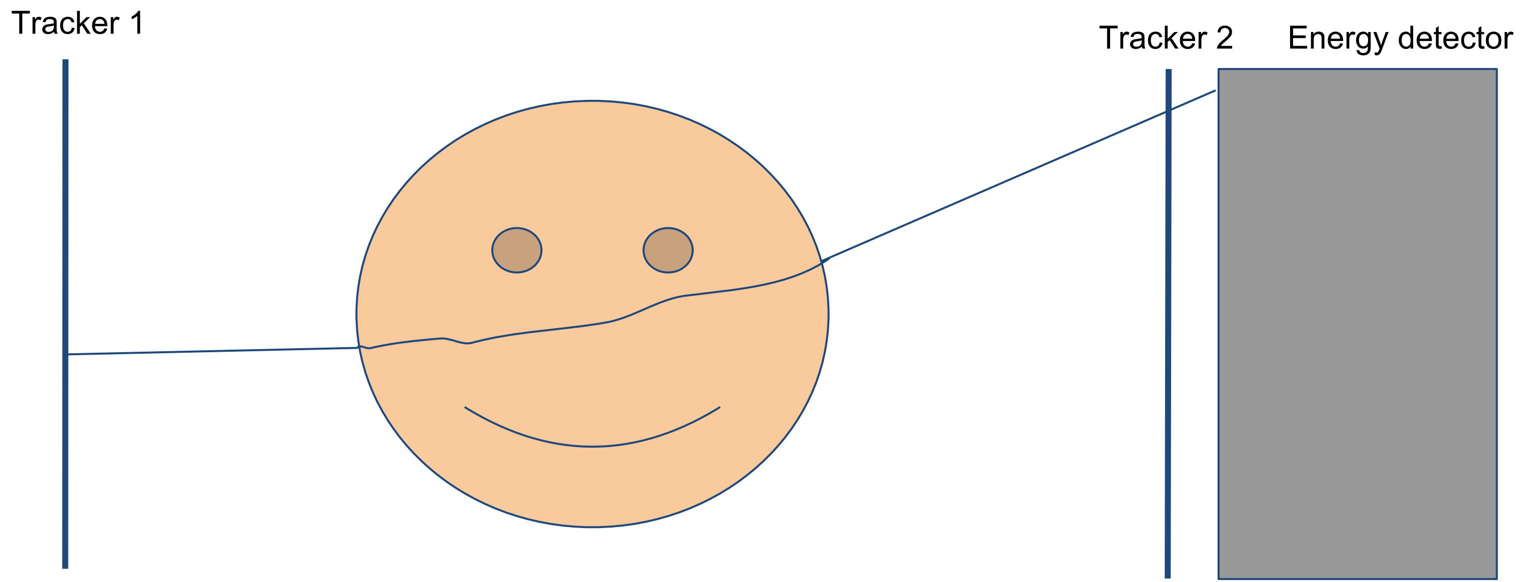
Monte-Carlo analysis of the effects of transverse heterogeneities on the most likely path of protons

Feriel Khellaf, Nils Krah, Ilaria Rinaldi, Jean Michel Létang, Simon Rit

Proton Imaging Workshop Lyon 2018

15/06/2018

Background



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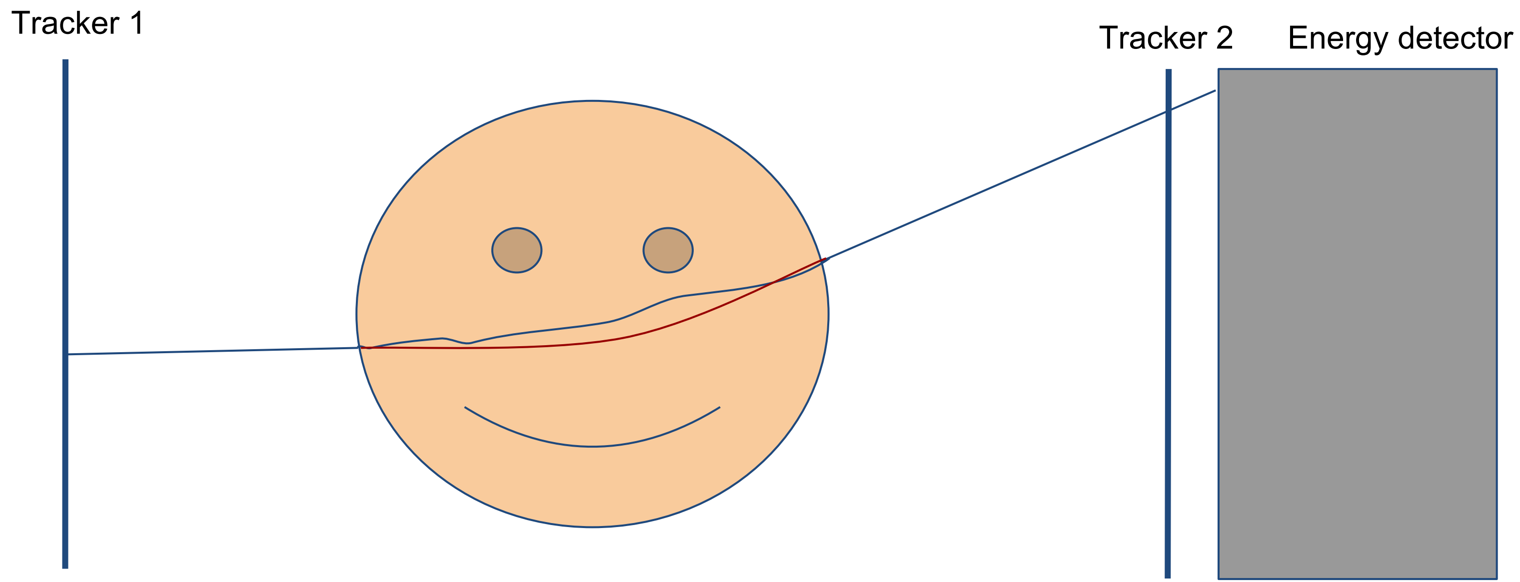

➔ Most likely path (MLP) (*Schulte et al., 2008*)

$$y_{\mathrm{MLP,theo}}(u) = (\Sigma_1^{-1} + R_1^T \Sigma_2^{-1} R_1)^{-1} (\Sigma_1^{-1} R_0 y_0 + R_1^T \Sigma_2^{-1} y_2)$$

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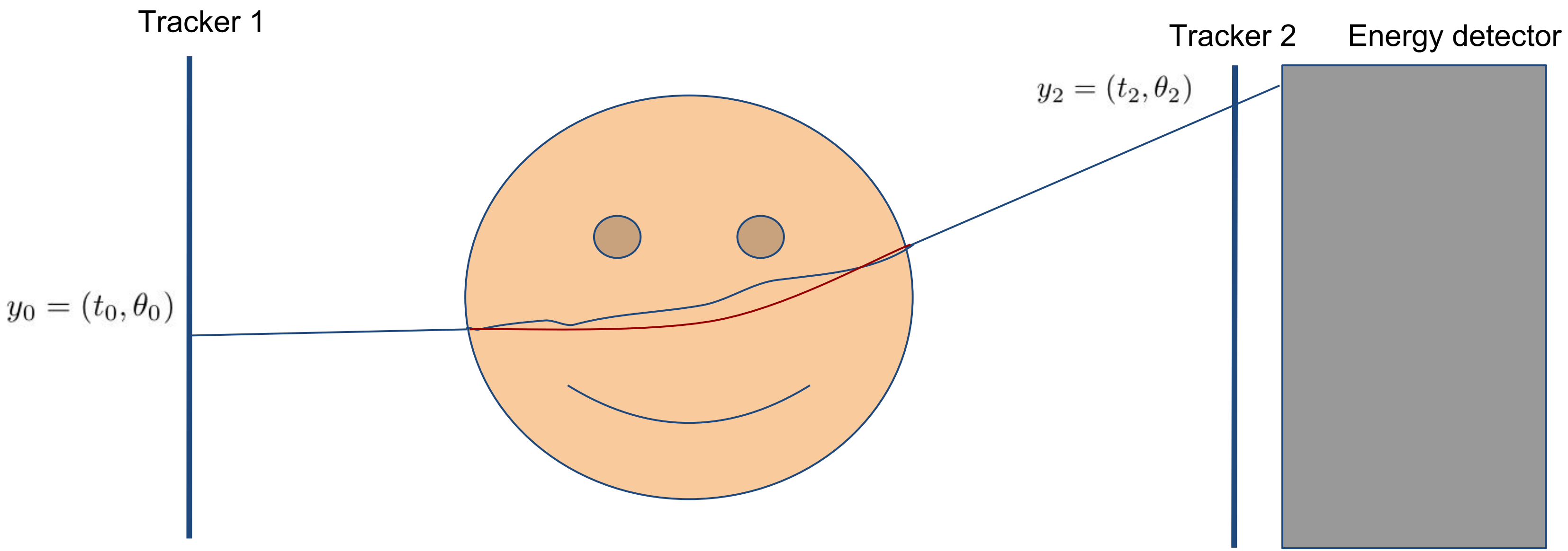

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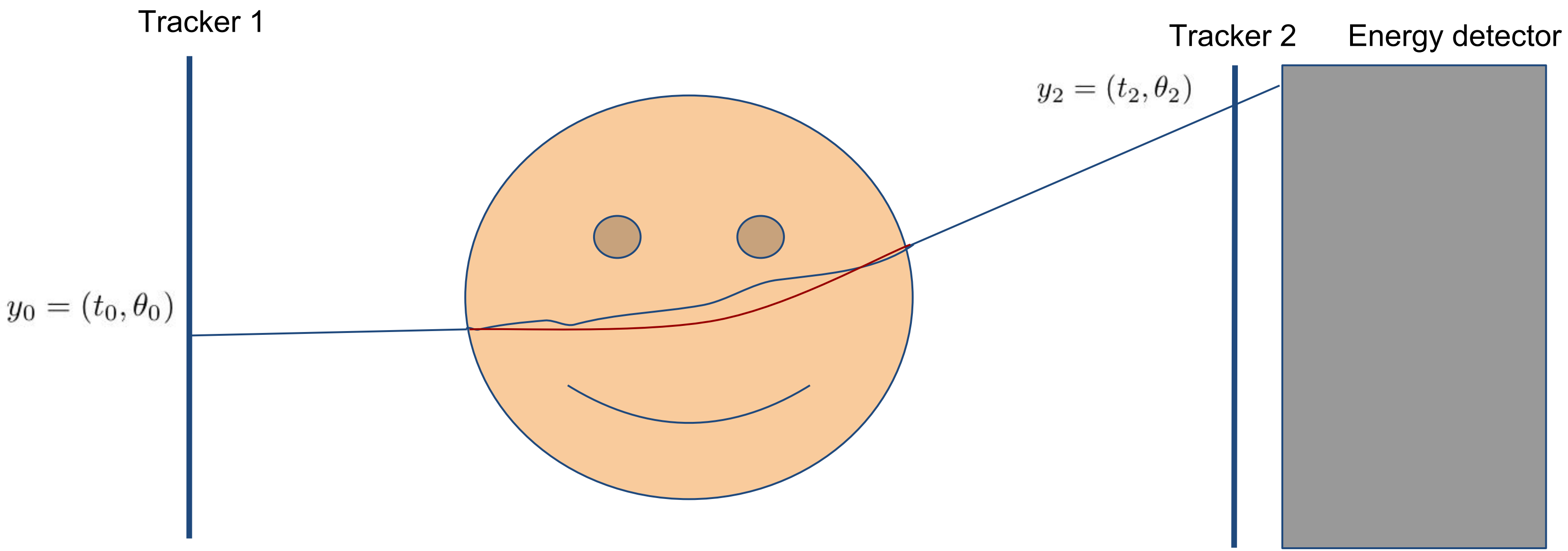

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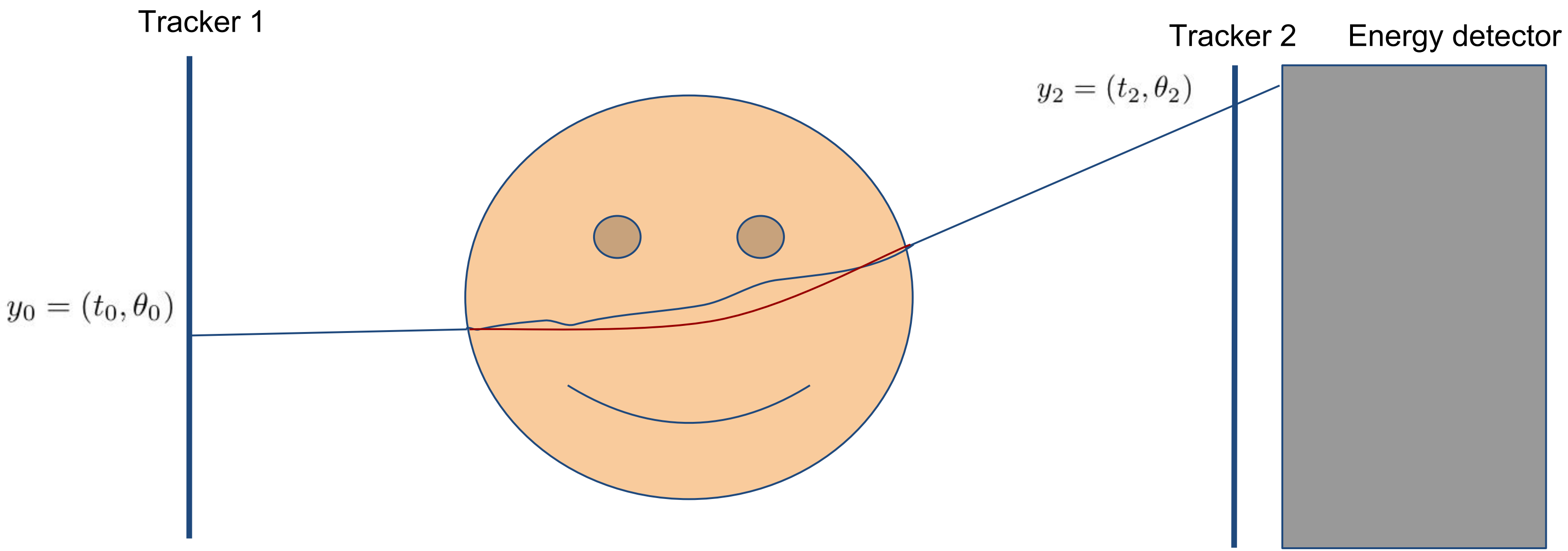

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Background

→ Homogeneous medium assumption in MLP formalism

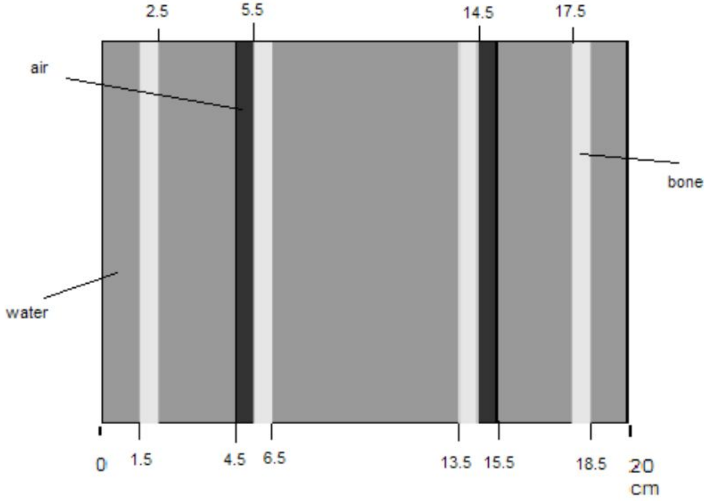
$$y_{\mathrm{MLP,theo}}(u) = (\Sigma_1^{-1} + R_1^T \Sigma_2^{-1} R_1)^{-1} (\Sigma_1^{-1} R_0 y_0 + R_1^T \Sigma_2^{-1} y_2)$$

- Radiation length X_0
- Momentum-velocity $\frac{1}{\beta(u)^2 p(u)^2}$
- Gaussian angular and spatial distribution (Fermi-Eyges theory)

Previous work

- ➔ Wong et al. : impact of longitudinal heterogeneities on the accuracy of the MLP

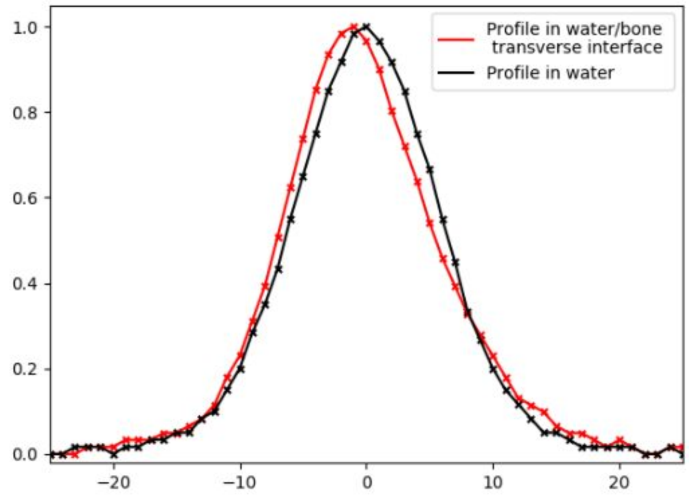
- ➔ Collins-Fekete et al. : extension of the MLP with prior-knowledge information on the tissues



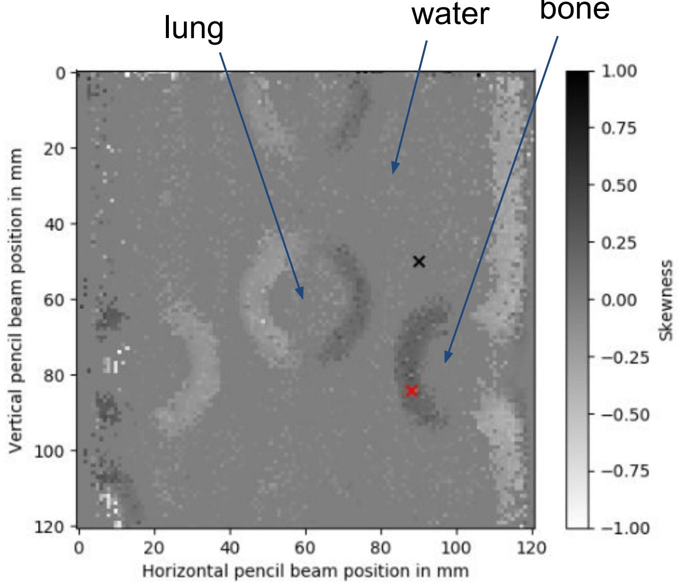
Heterogeneous phantom (Wong et al.)

Previous work

➔ Experimental observations on CIRS electron density phantom



Fluence profiles

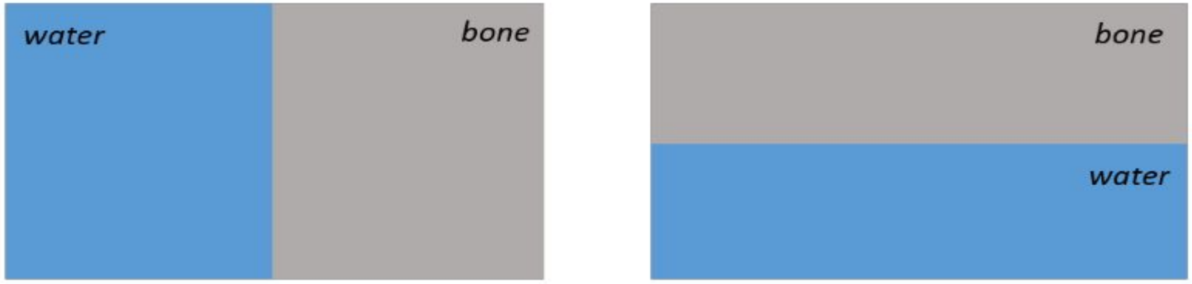


Skewness map

Objective

Homogeneous medium assumption in MLP formalism

Investigate the effects of transverse heterogeneities on the accuracy of the MLP



Longitudinal heterogeneity | Transverse heterogeneity

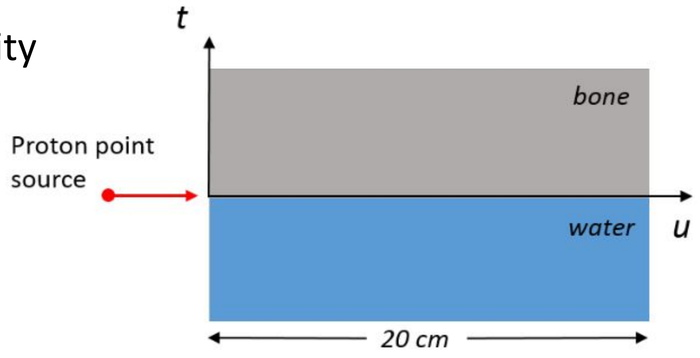
Methods

➔ Analytical MLP

$$y_{\mathrm{MLP,theo}}(u) = (\Sigma_1^{-1} + R_1^T \Sigma_2^{-1} R_1)^{-1} (\Sigma_1^{-1} R_0 y_0 + R_1^T \Sigma_2^{-1} y_2)$$

➔ "Real" MLP from MC simulations

1) Simulate a transverse heterogeneity in GATE/Geant4



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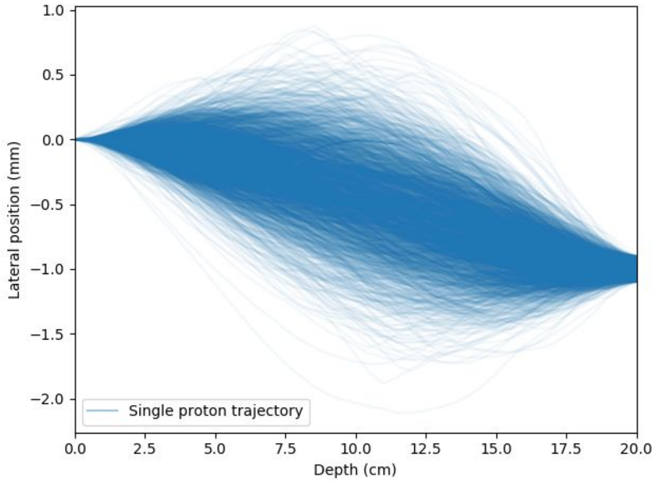
1) Simulate a transverse heterogeneity in GATE/Geant4
2) Filter protons to mimic the single tracking setup



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1) Simulate a transverse heterogeneity in GATE/Geant4
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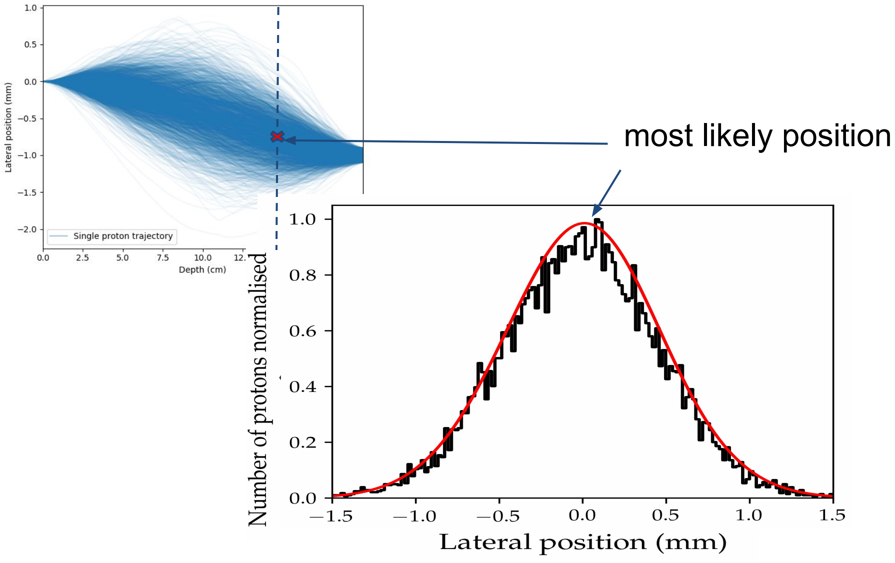

Methods

→ Analytical MLP

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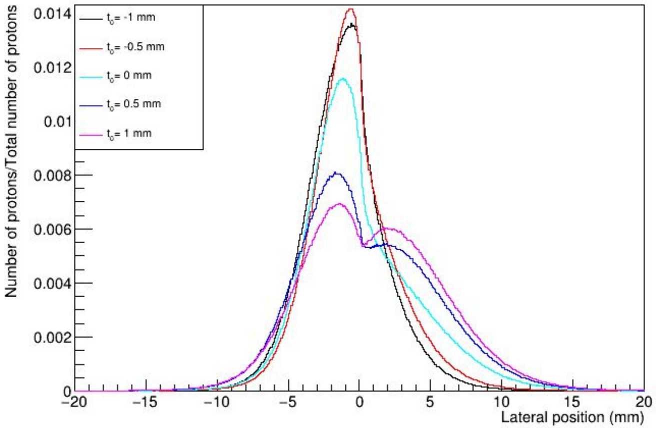
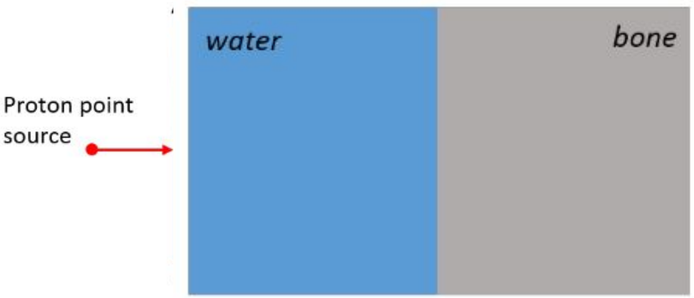
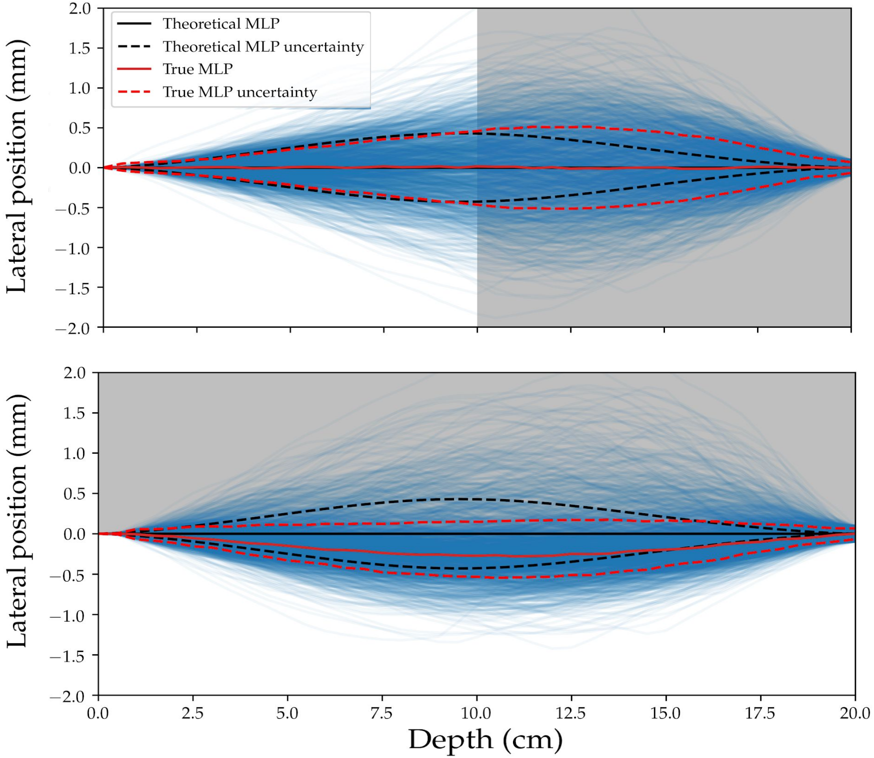


→ Histogram fit : $f(x) = \frac{A_1}{\sigma_1\sqrt{2\pi}} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + \frac{A_2}{\sigma_2\sqrt{2\pi}} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}$

Results & discussion





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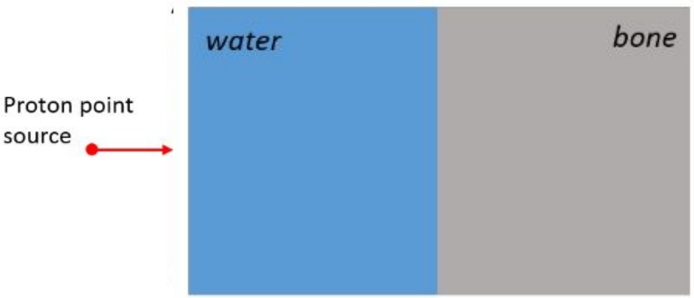
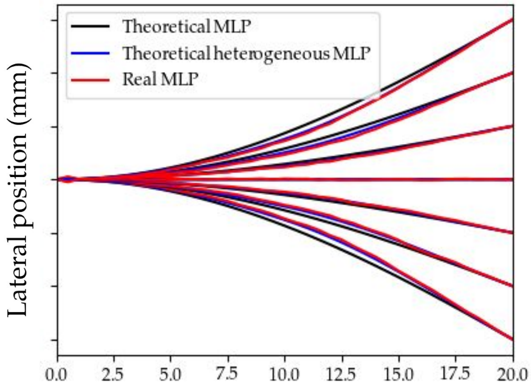
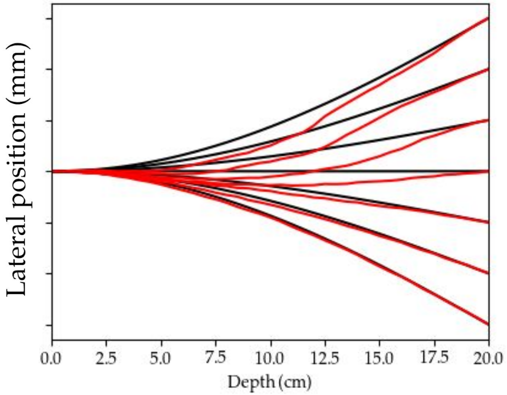


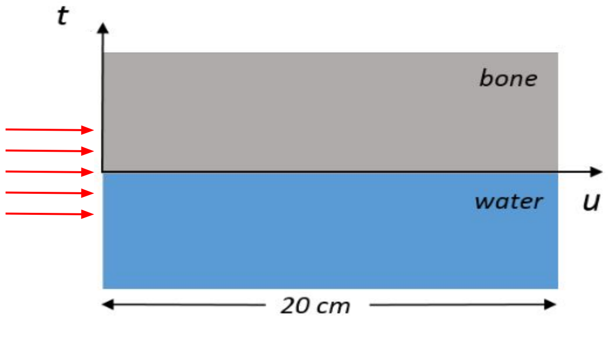


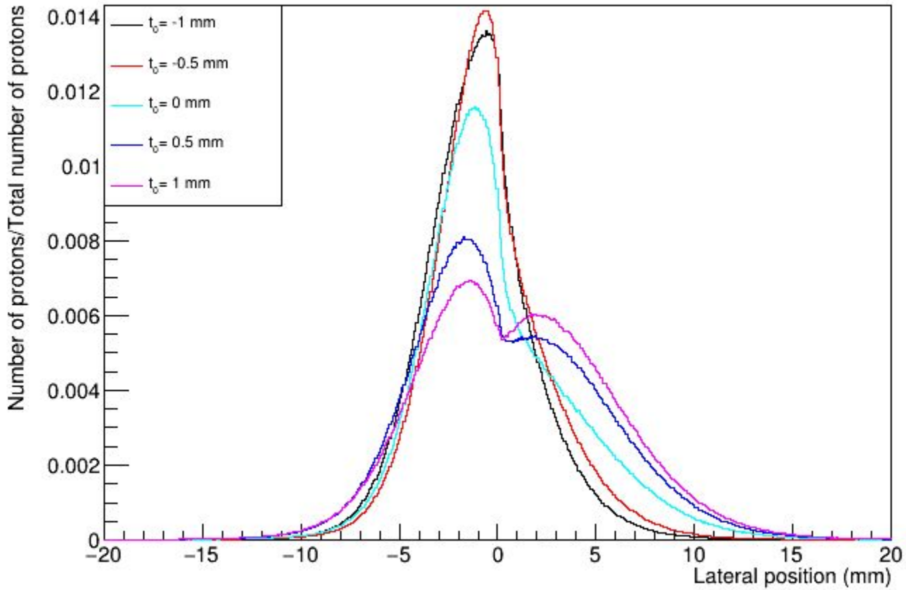


➔ max difference between theoretical and real MLP ≈ 0.5 mm

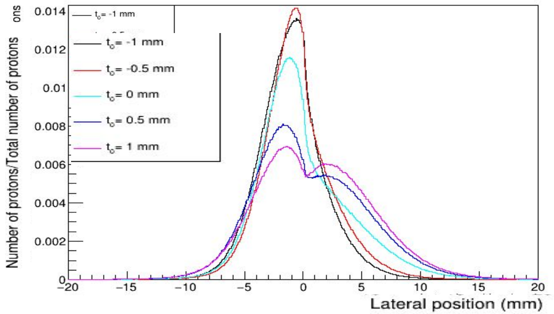
Results & discussion

➔ Non Gaussian spatial and angular distributions





Results & discussion



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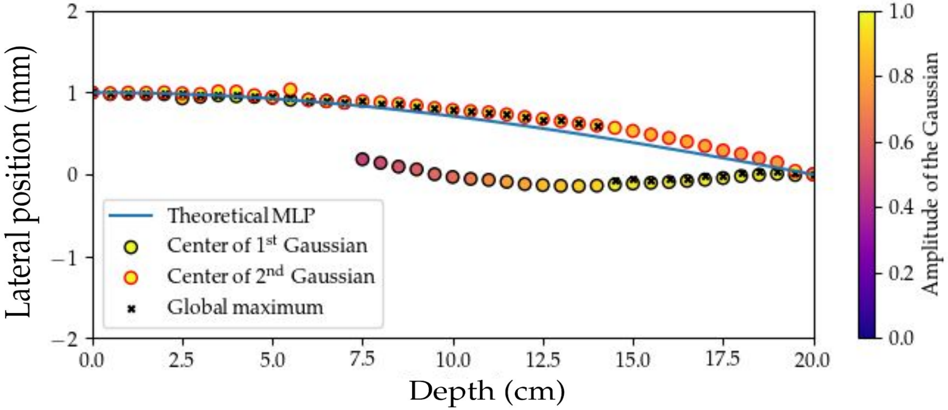
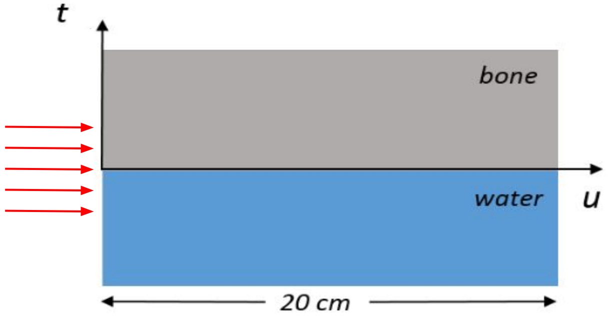
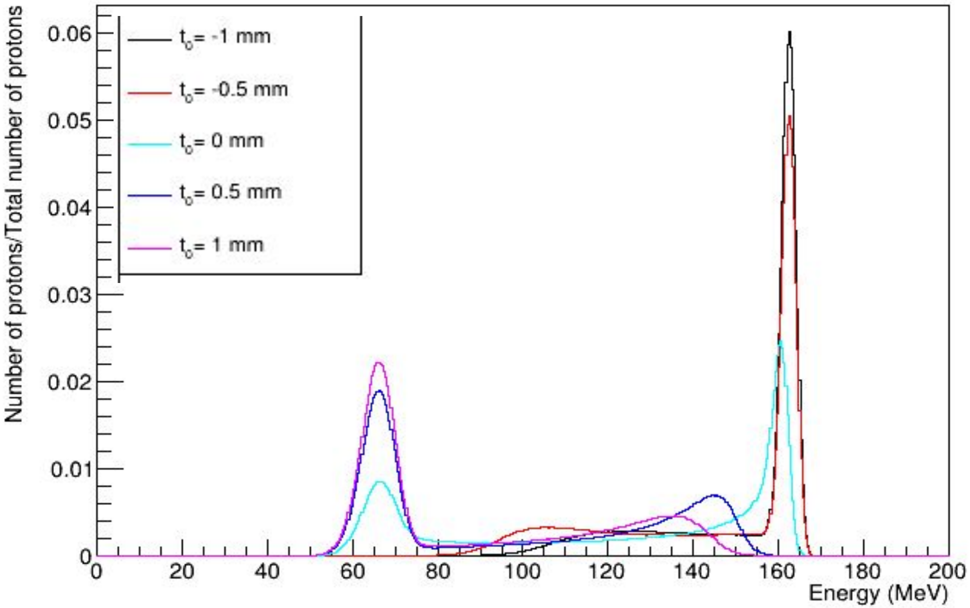


Results & discussion

➔ Exit energy profiles





Conclusion

- ➢ Study of a worst case scenario
- ➢ Maximum error of 0.5 mm close to the MLP formalism's uncertainty (0.43 mm)

The current MLP formalism is accurate enough even in the presence of transverse heterogeneities

Thank you for your attention!